Menoufia University

Faculty of Electronic Engineering

MECHATRONIC-3

Kinematics Fundamentals II

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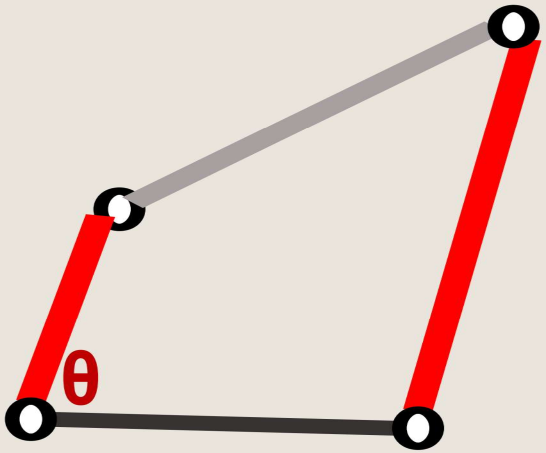
Outline

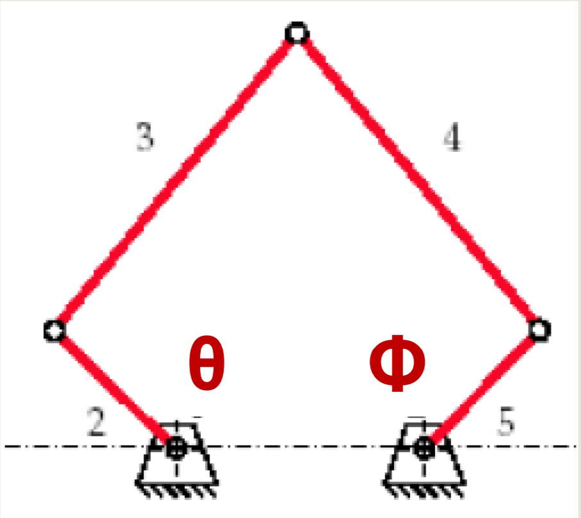
- ❑ **Mobility**
- ❑ **Number Synthesis**
- ❑ **Isomers**
- ❑ **Inversions**
- ❑ **Grashof Condition**

Mobility

Mobility (the number of degrees of freedom) of a mechanism: is the minimum number of independent coordinates needed to specify uniquely the position of the mechanism.



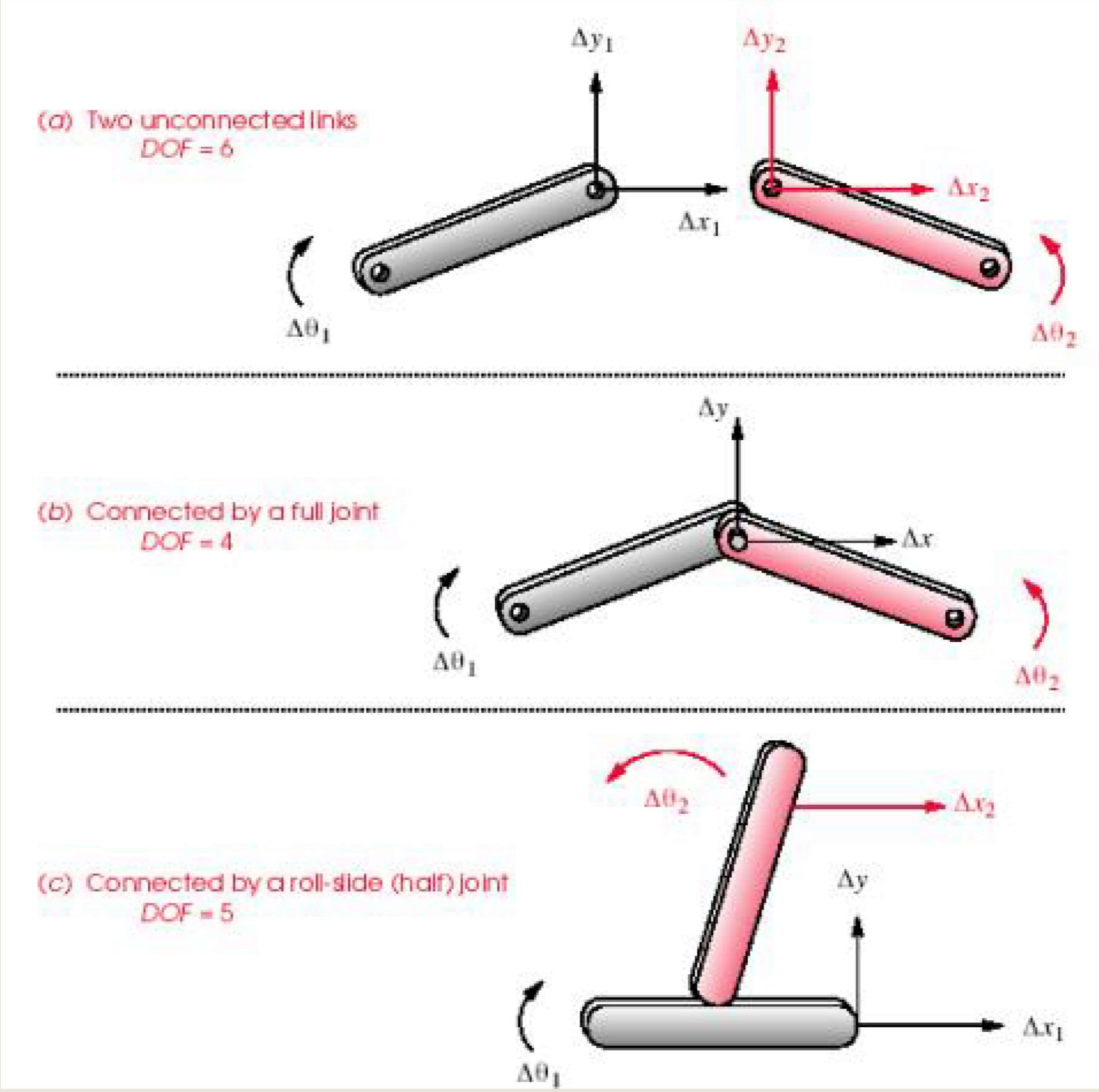
One needs one variable **θ** to specify the mechanism position (or to draw it). The mechanism mobility equals 1.



One needs two variable **θ, Φ** to specify the mechanism position (or to draw it). The mechanism mobility equals 2.

Mobility

Joints Reduce System DOF



Determining Mobility

Grubler & Kutzbach Equations

$$M = 3L - 2J - 3G$$

where:

M = degree of freedom or mobility

L = number of links

J = number of joints

G = number of grounded links

- Note that in any real mechanism, even if more than one link of the kinematic chain is grounded, the net effect will be to create one larger ground link. Thus G is always one, and Gruebler's equation becomes:

$$M = 3(L - 1) - 2J$$

Determining Mobility

Grubler & Kutzbach Equations

- The value of J in previous equations must reflect the value of all joints in the mechanism. That is, half joints count as 1/2 because they only remove one DOF.

- It is less confusing if we use kutzbach's modification of Gruebler's equation in this form:

$$\boxed{M = 3(L - 1) - 2J_1 - J_2}$$

where:

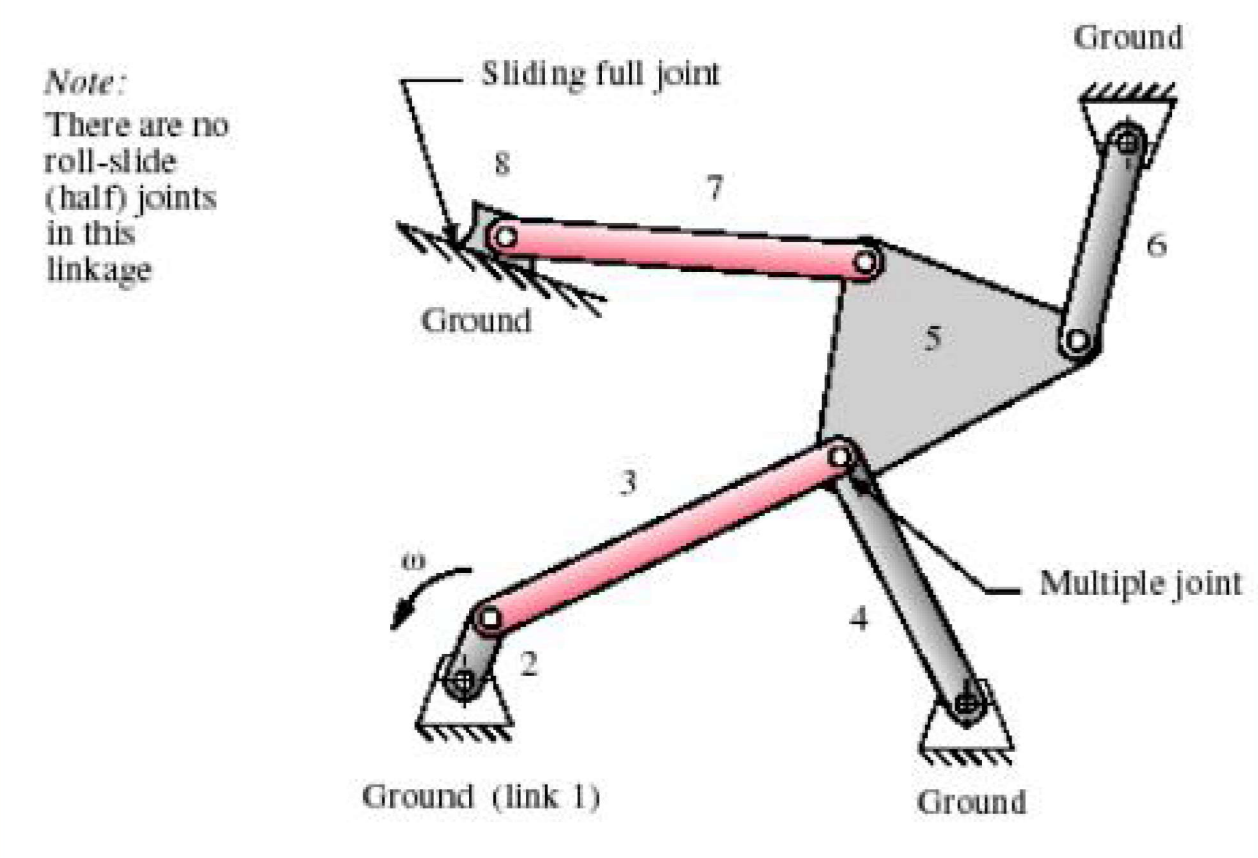
M = degree of freedom or mobility

L = number of links

J_1 = number of 1DOF (full) joints

J_2 = number of 2 DOF (half) joints

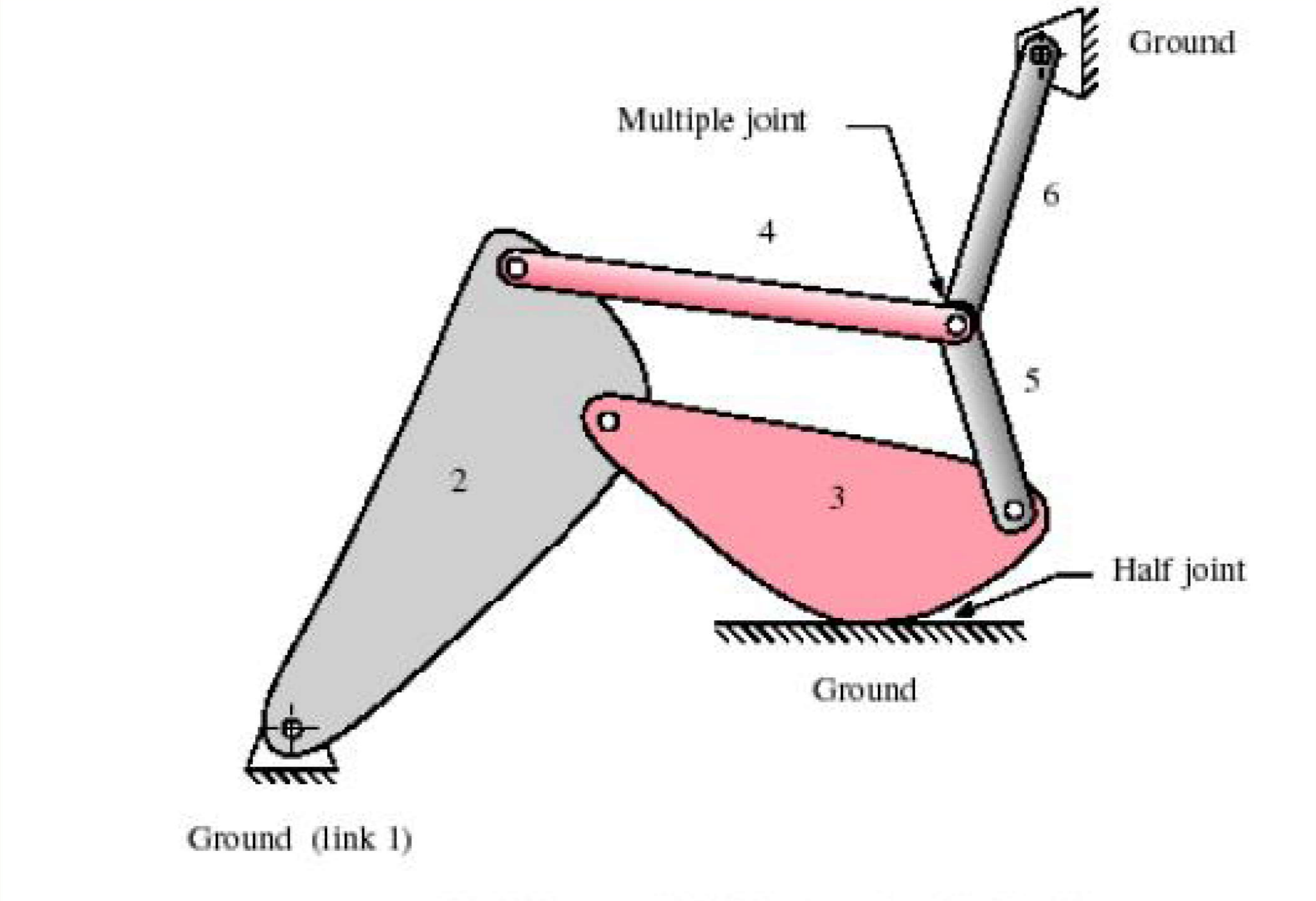
Applying Mobility Equations



(a) Linkage with full and multiple joints

$$\mathbf{M = 3\,(L - 1) - 2\,J_1 - J_2} = 3(8-1) - 2\text{x}10 - 0 = 1$$

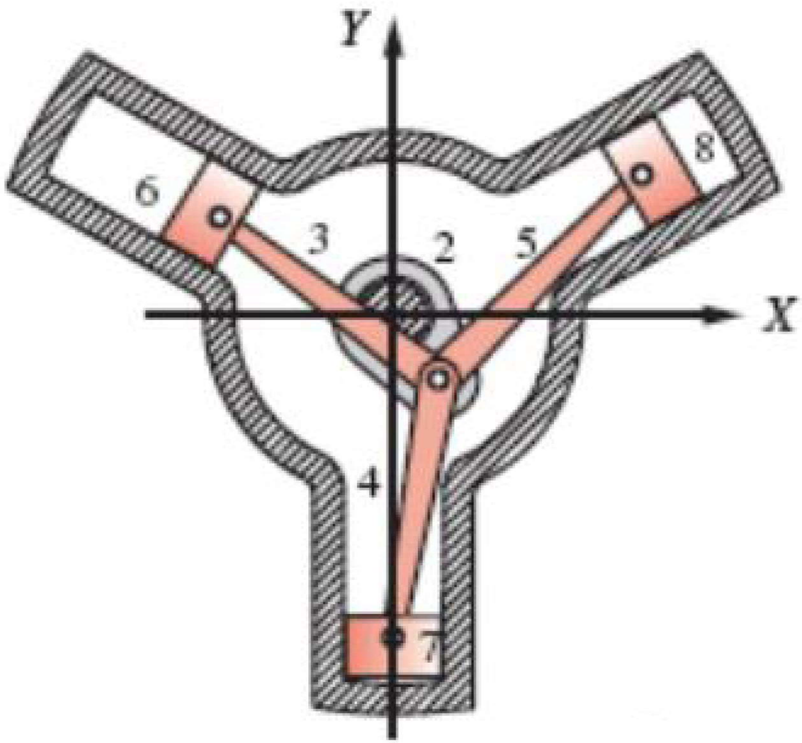
Applying Mobility Equations



(b) Linkage with full, half, and multiple joints

$$\mathbf{M = 3\ (L - 1) - 2\ J_1 - J_2} = 3(6\text{-}1) - 2\text{x}7 - 1 = 0$$

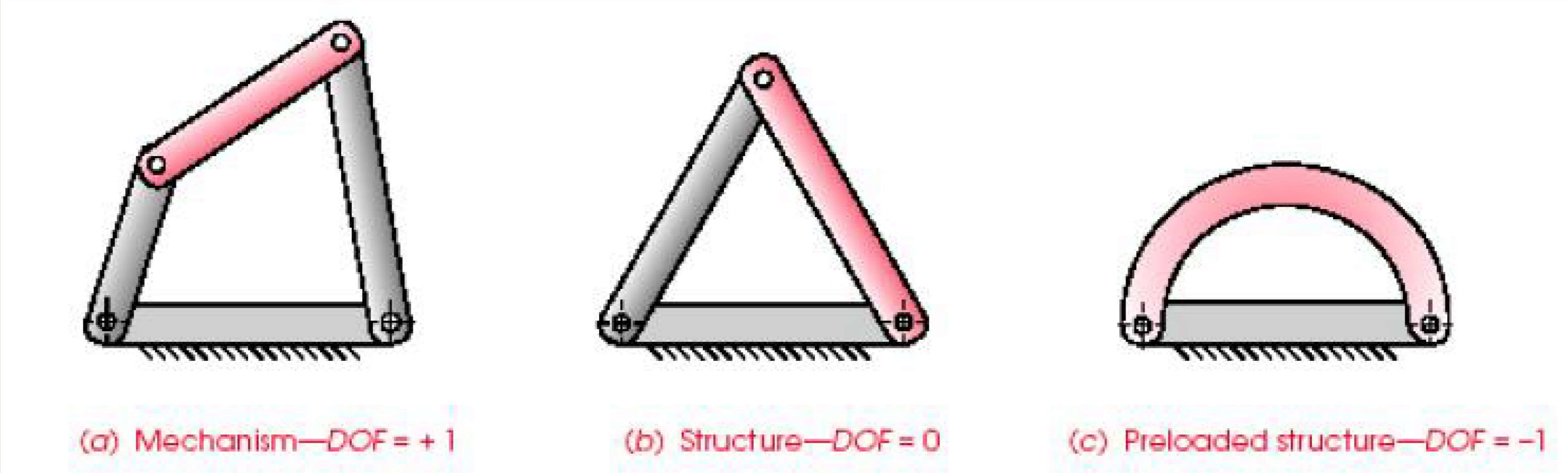
Applying Mobility Equations



(c) Radial compressor

$$\mathbf{M = 3\,(L-1) - 2\,J_1 - J_2} = 3(8\text{-}1) - 2\text{x}10 - 0 = 1$$

Mechanisms and Structures



- The degree of freedom of an assembly of links completely predicts its character. There are only three possibilities.

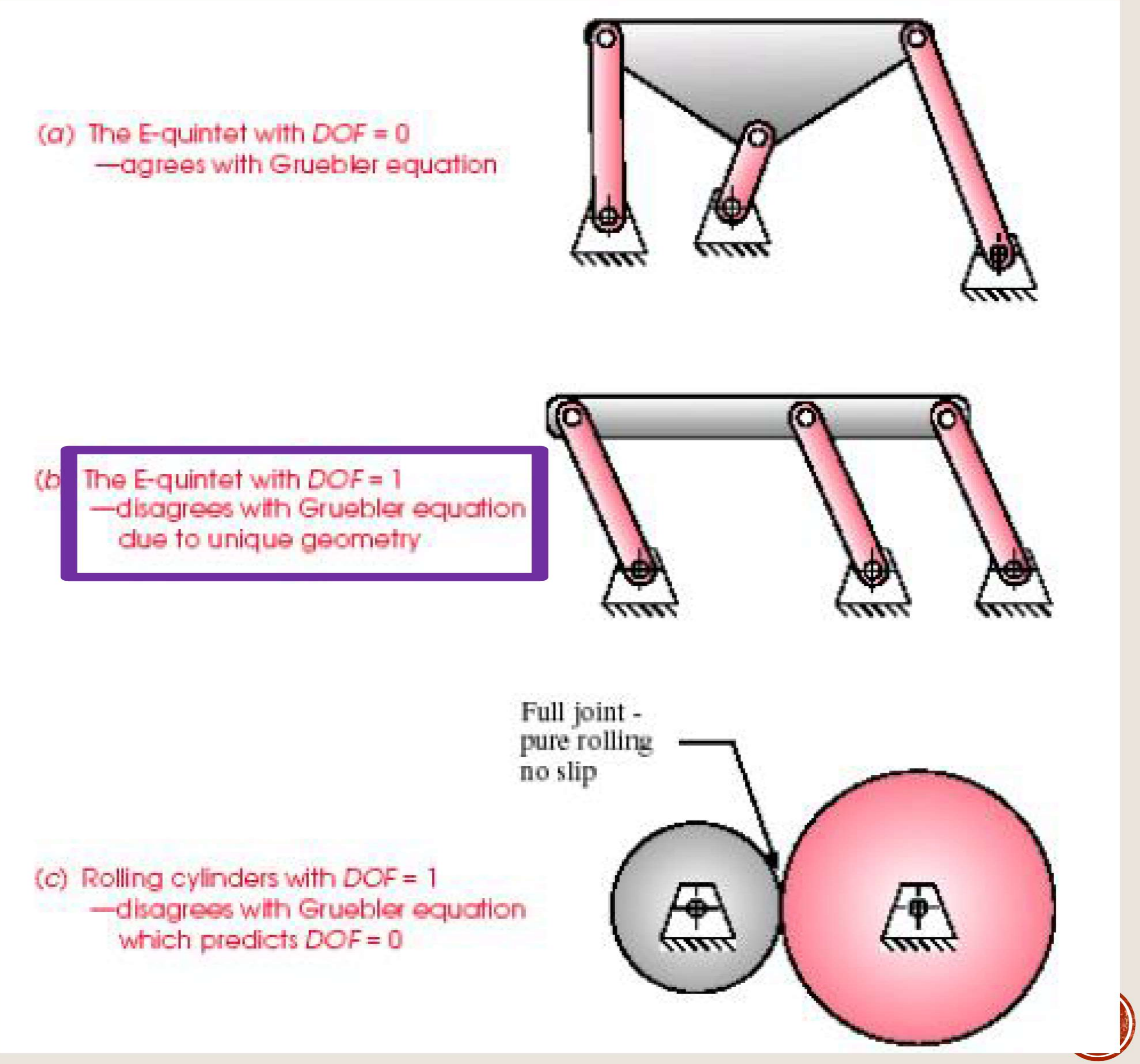
  a) M > 0 → **mechanism**, links will have relative motion.

  b) M = 0, → **structure**, no relative motion between links is possible.

  c) M < 0, → **preloaded structure**, no relative motion between links is possible and some stresses may be present.

Gruebler Paradoxes

- Because the Gruebler criterion pays no attention to link sizes or shapes, it can give misleading results in the face of unique geometric configurations.



Number Synthesis

- Number Synthesis means the determination of the number and order of links and joints necessary to produce motion of a particular DOF.

TABLE 2-2 1-*DOF* Planar Mechanisms with Revolute Joints and Up to 8 Links

Total Links	Link Sets				
	Binary	Ternary	Quaternary	Pentagonal	Hexagonal
4	4	0	0	0	0
6	4	2	0	0	0
6	5	0	1	0	0
8	7	0	0	0	1
8	4	4	0	0	0
8	5	2	1	0	0
8	6	0	2	0	0
8	6	1	0	1	0

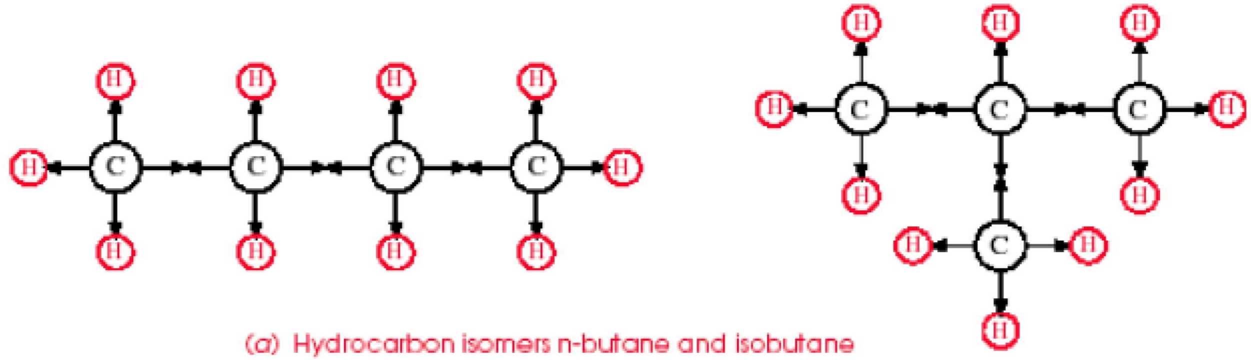
- Some combinations will make valid "Isomers," and some will not.

Isomers

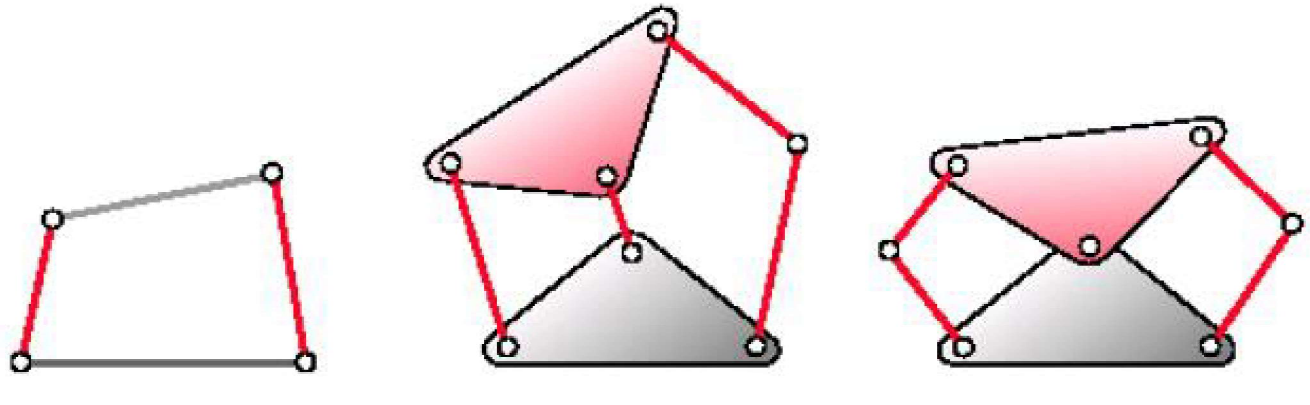
Isomers in chemistry are compounds that have the same number and type of atoms but which are interconnected differently and thus have different physical properties.

Linkage isomers are analogous to these chemical compounds.

C_4H_{10}



(a) Hydrocarbon isomers n-butane and isobutane



The only fourbar isomer

Stephenson's sixbar isomer

Watt's sixbar isomer

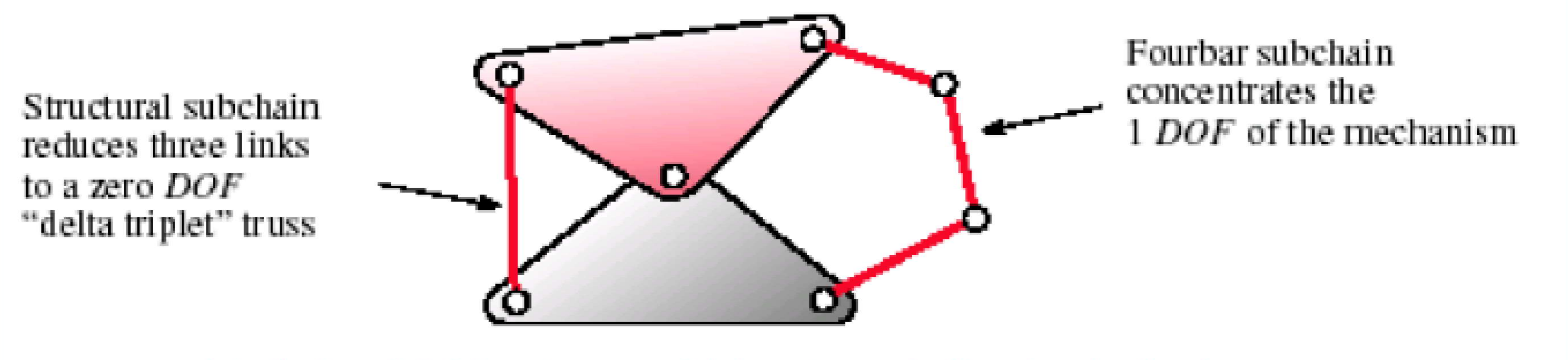
(b) All valid isomers of the fourbar and sixbar linkages

Number of Valid Isomers

Links	Valid Isomers
4	1
6	2
8	16
10	230
12	6856 or 6862*

* Researchers disagree.

Invalid Isomers



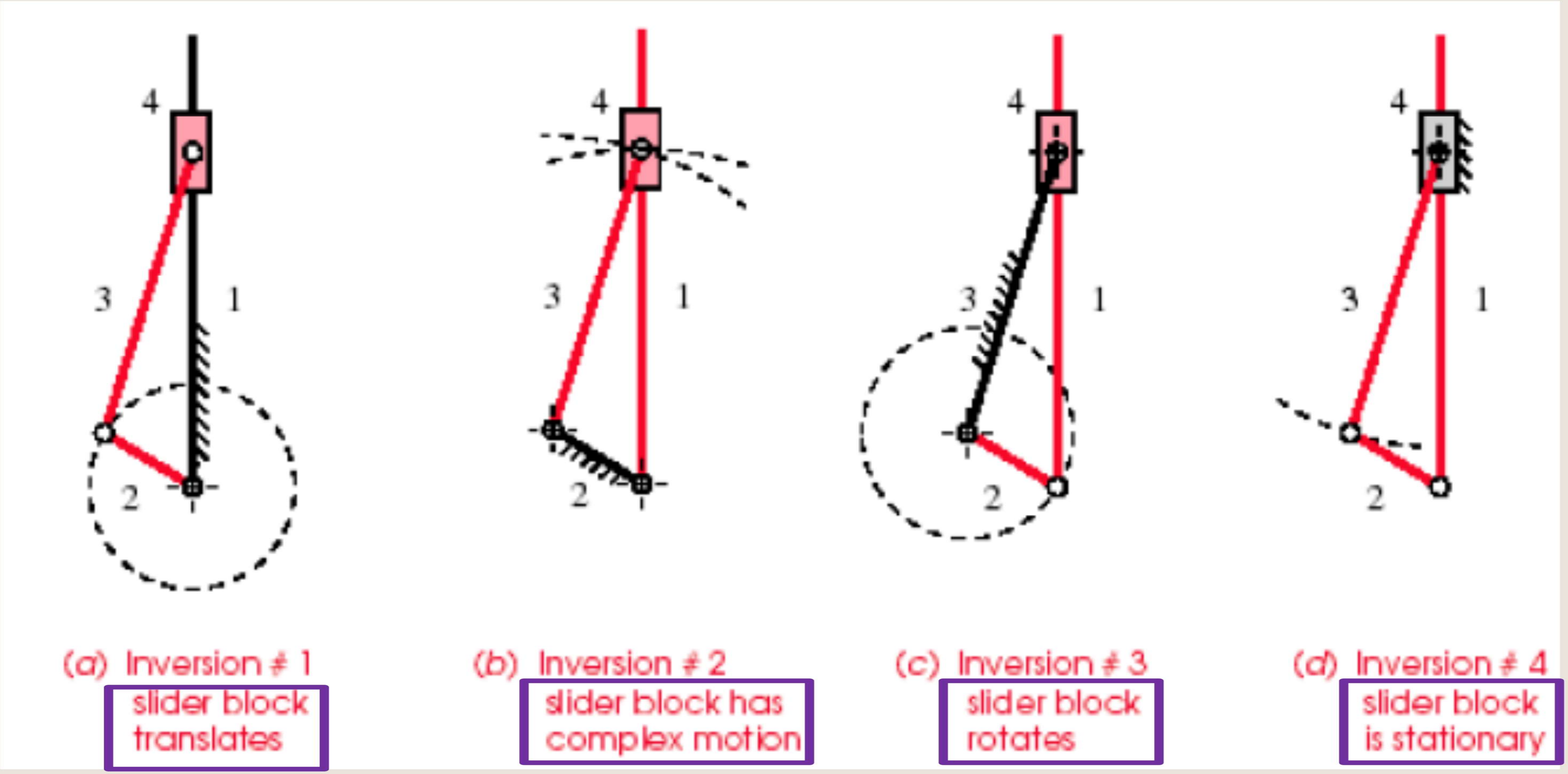
(c) An invalid sixbar isomer which reduces to the simpler fourbar

- The structural subchain of the two ternaries and the single binary effectively reduced to a structure that acts like a single link.
- Thus this arrangement has been reduced to the simpler case of the fourbar linkage despite its six bars.
- This is an invalid isomer and is rejected.

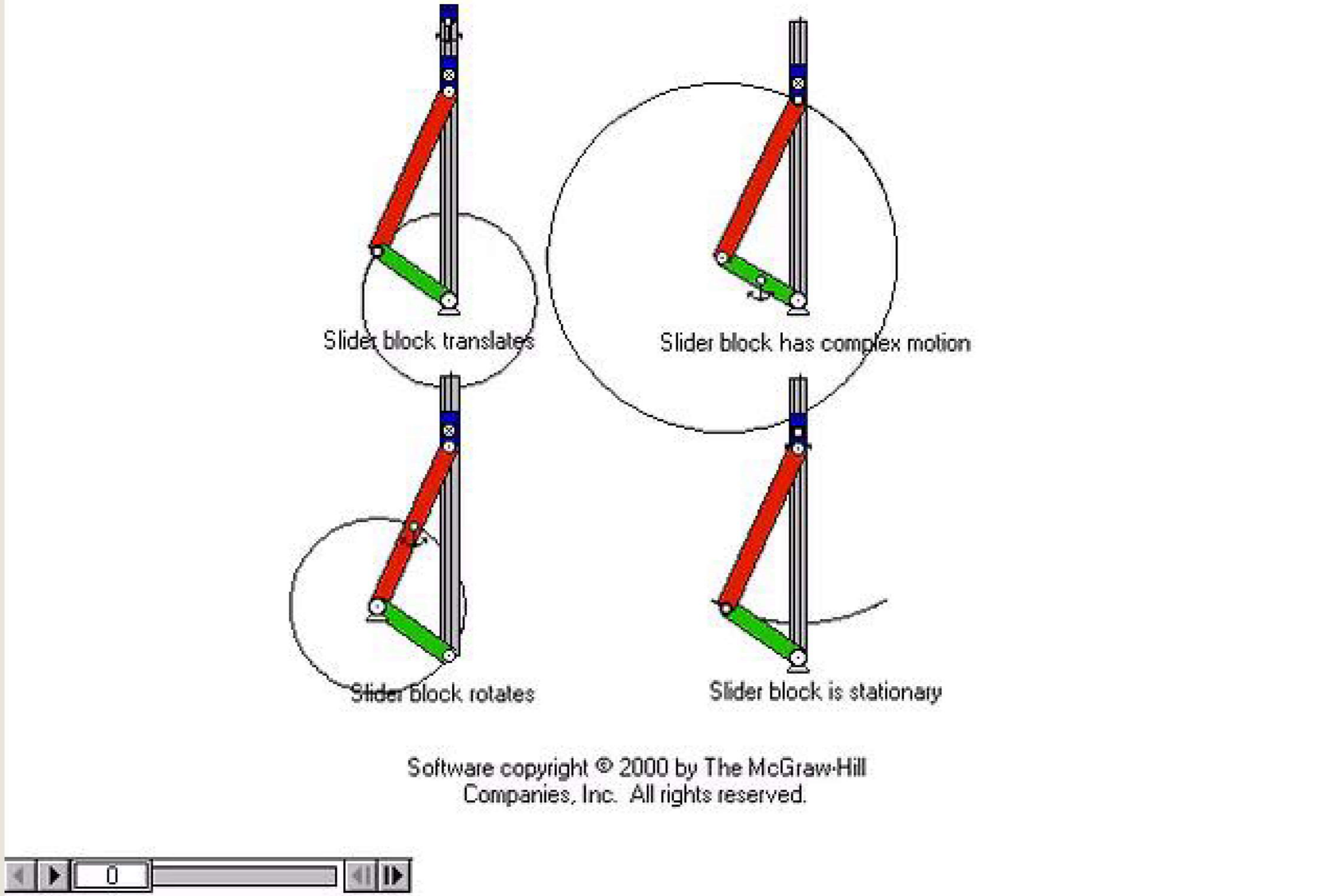
Inversions

- An inversion is created by grounding a different link in the kinematic chain.
- Thus, there are as many inversions of a given chain as it has links.
- The motions resulting from each inversion can be quite different.
- But some inversions of a linkage may yield motions similar to other inversions of the same linkage.

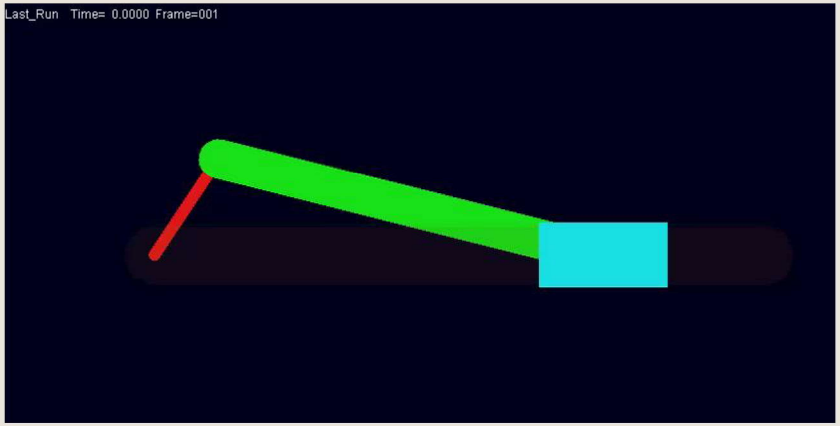
Inversions of the Slider Crank



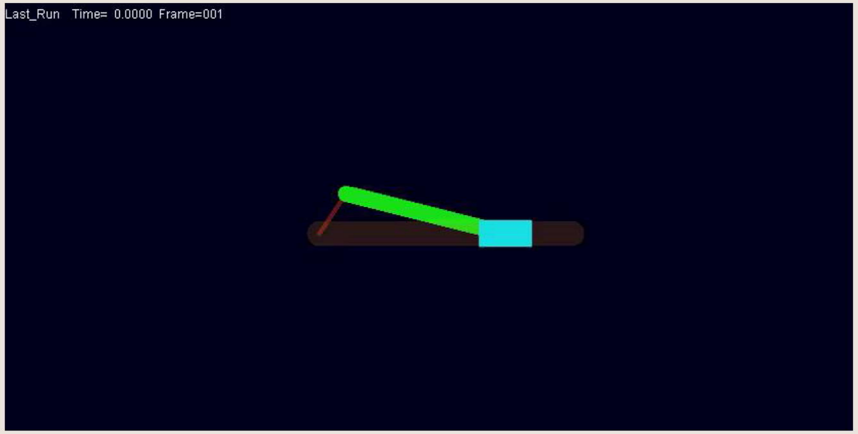
Inversions of the Slider Crank



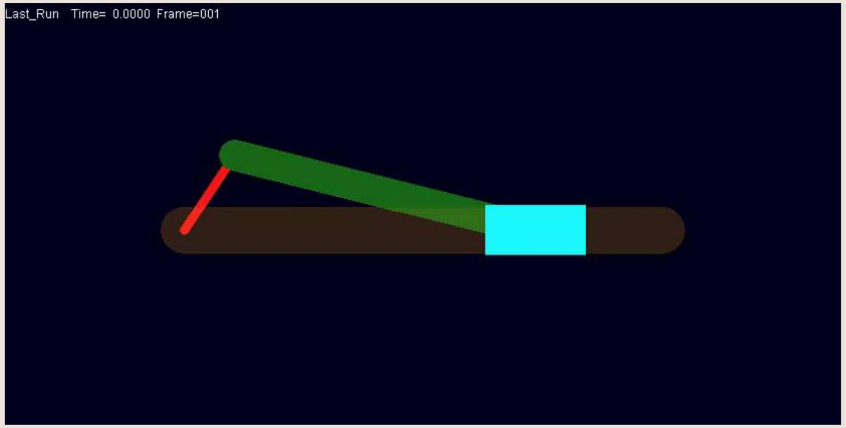
Inversions of the Slider Crank



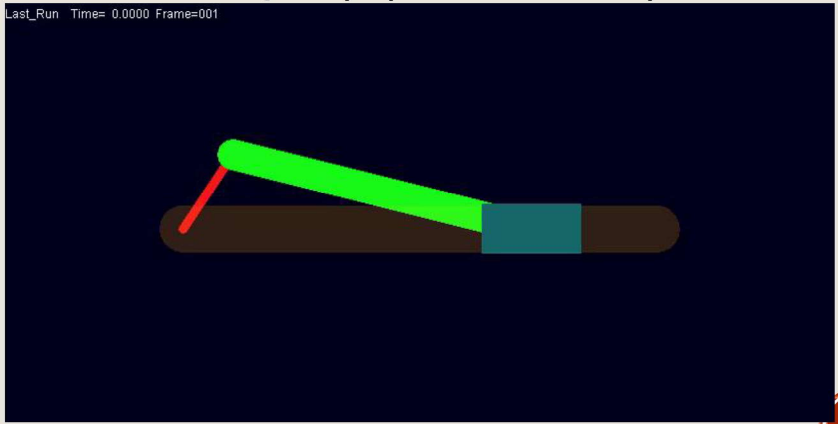
Used in engines and piston pumps



Quick-return motion mechanism (Crank-shaper) (Whitworth)

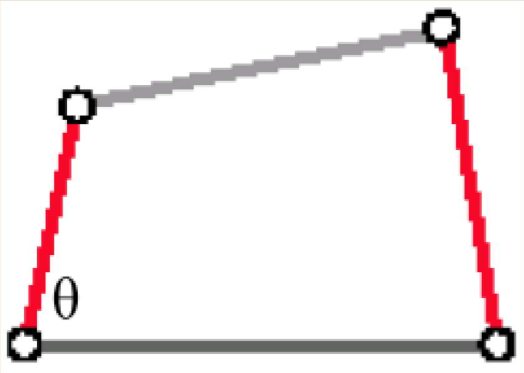


Slider block rotates(Oscillating cylinder engine)



Hand operated pump

Grashof Condition of the Four-bar Mechanism



- ***Four bar mechanism*** is in fact the most common and ubiquitous device used in machinery.

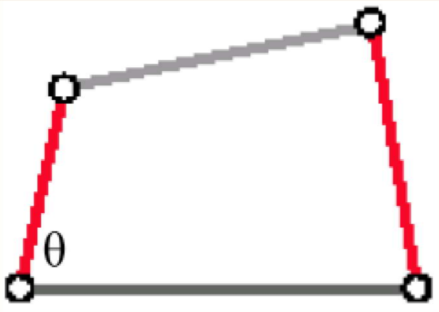
- It is also extremely versatile in terms of the types of motion that it can generate.

- The ***Grashof condition*** is a very simple relationship that predicts the rotation behavior or rotatability of a fourbar linkage's inversions based only on the link lengths.

Grashof Condition of the Four-bar Mechanism

Let:

S = length of shortest link
L = length of longest link
P = length of one remaining link
Q = length of other remaining link



Then if :

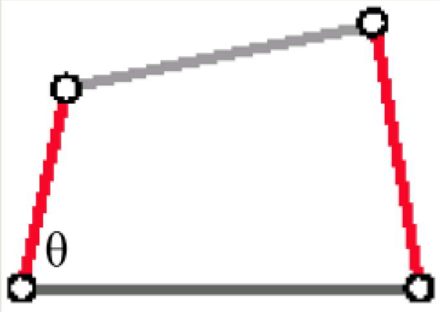
$$\boxed{S + L \leq P + Q}$$

- The linkage is Grashof and at least one link will be capable of making a full revolution with respect to the ground plane. This is called a ***Class I*** kinematic chain.

Grashof Condition of the Four-bar Mechanism

Then if :

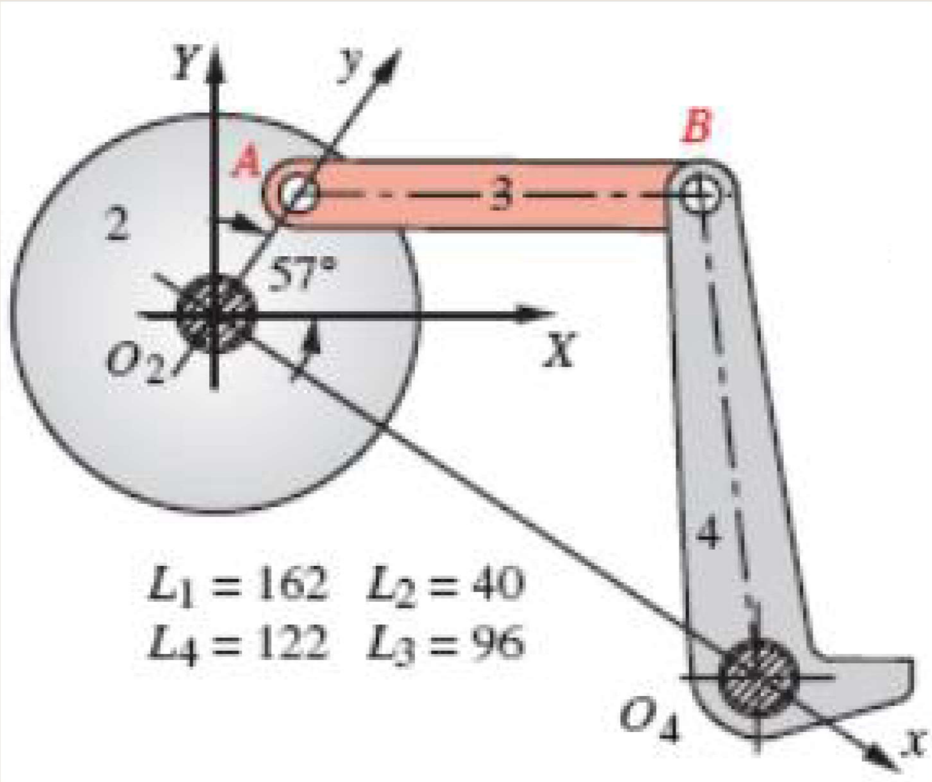
$$\boxed{S + L > P + Q}$$

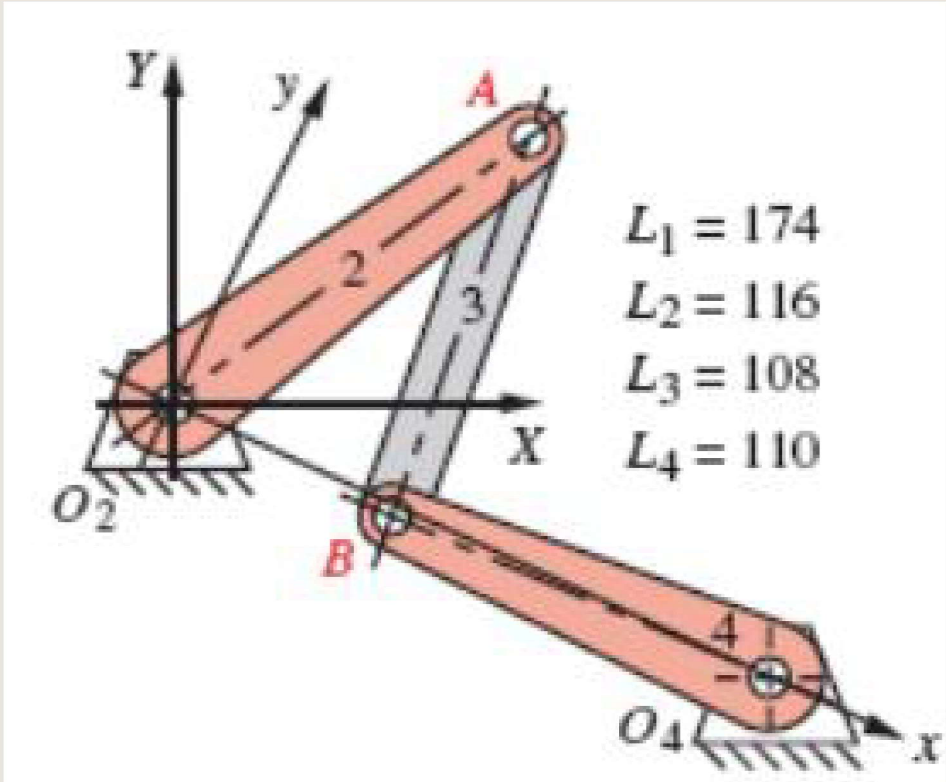


- Then the linkage is non-Grashof and no link will be capable of a complete revolution relative to any other link. This is a ***Class II*** kinematic chain.

  - The motions possible from a fourbar linkage will depend on both the Grashof condition and the inversion chosen.

Find Their Grashof Conditions

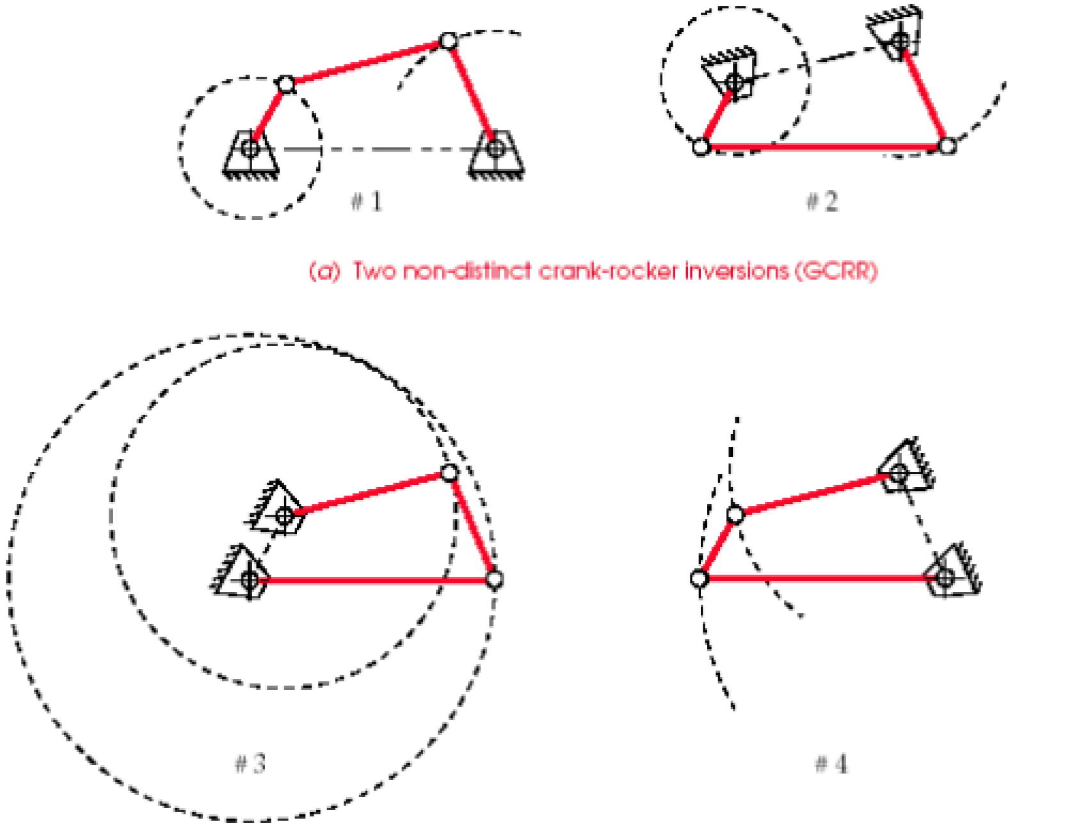




Inversions of the Grashof Fourbar

For the ***Class I*** *case,* $S + L < P + Q$

- The motions possible from a fourbar linkage will depend on both the Grashof condition and the inversion chosen.



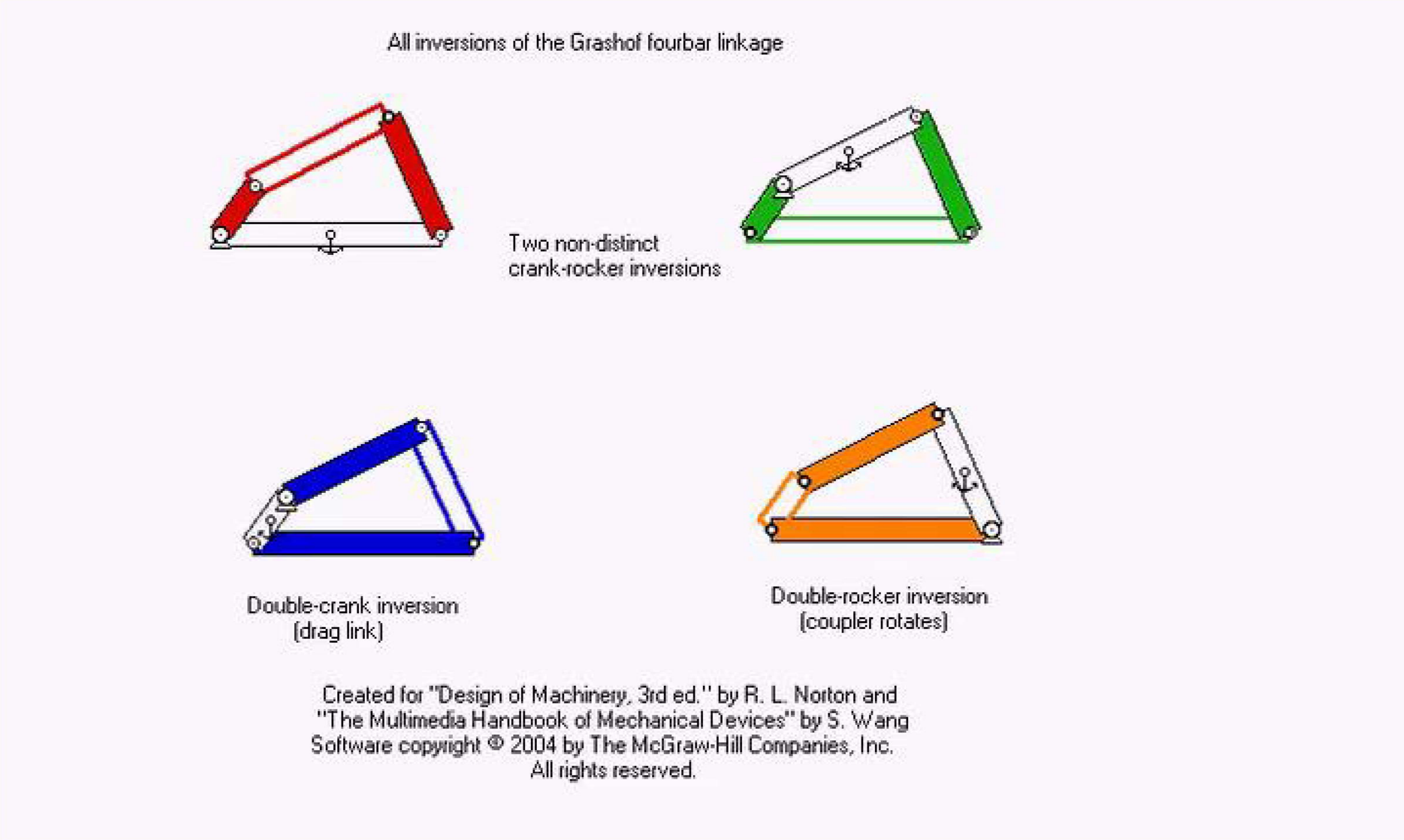
(a) Two non-distinct crank-rocker inversions (GCRR)

(b) Double-crank inversion (GCCC) (drag link mechanism)

(c) Double-rocker inversion (GRCR) (coupler rotates)

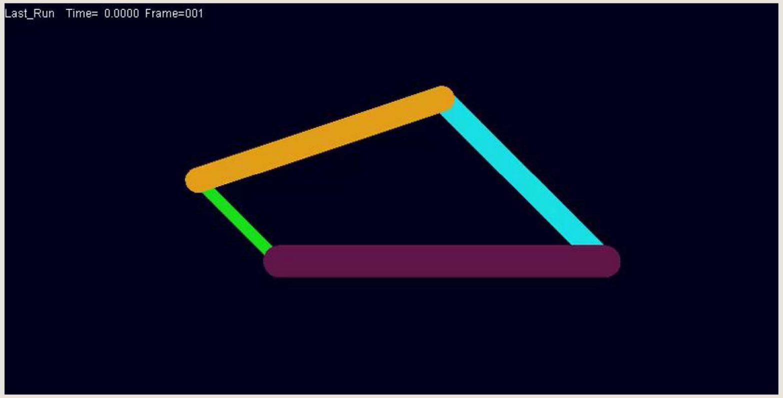
Inversions of the Grashof Fourbar

*For the **Class I** case,* $S + L < P + Q$

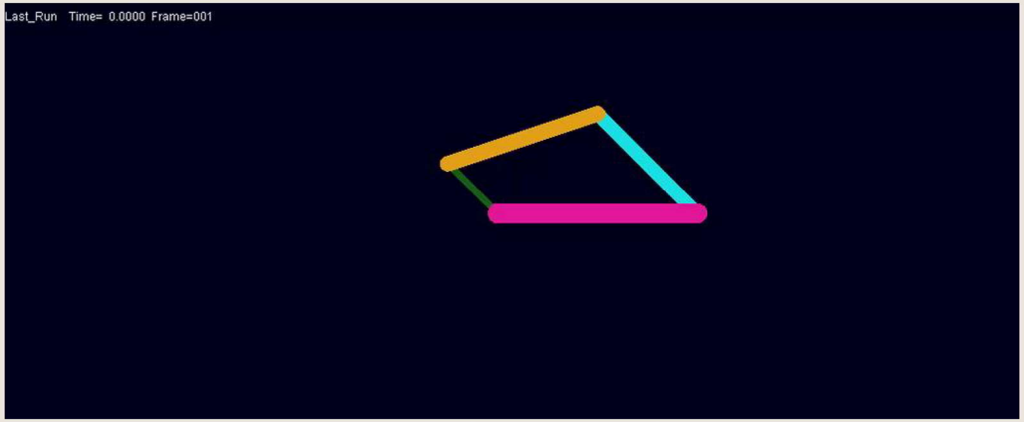


Inversions of the Grashof Fourbar

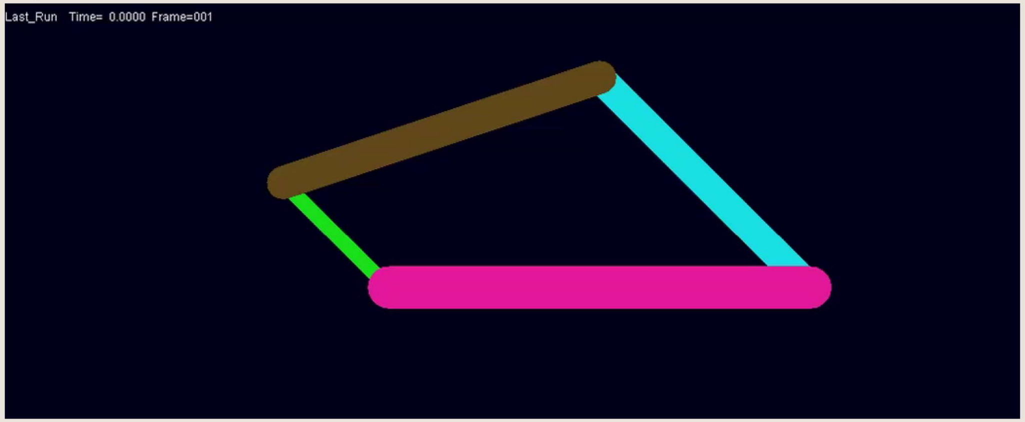
*For the **Class I** case,* $S + L < P + Q$



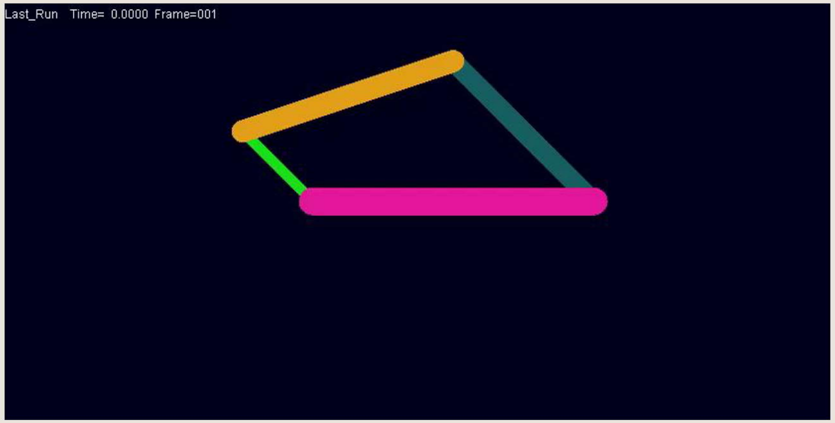
Crank – rocker
(a link attached to the shortest link is fixed)



Double crank (drag link)
(the shortest link is fixed)



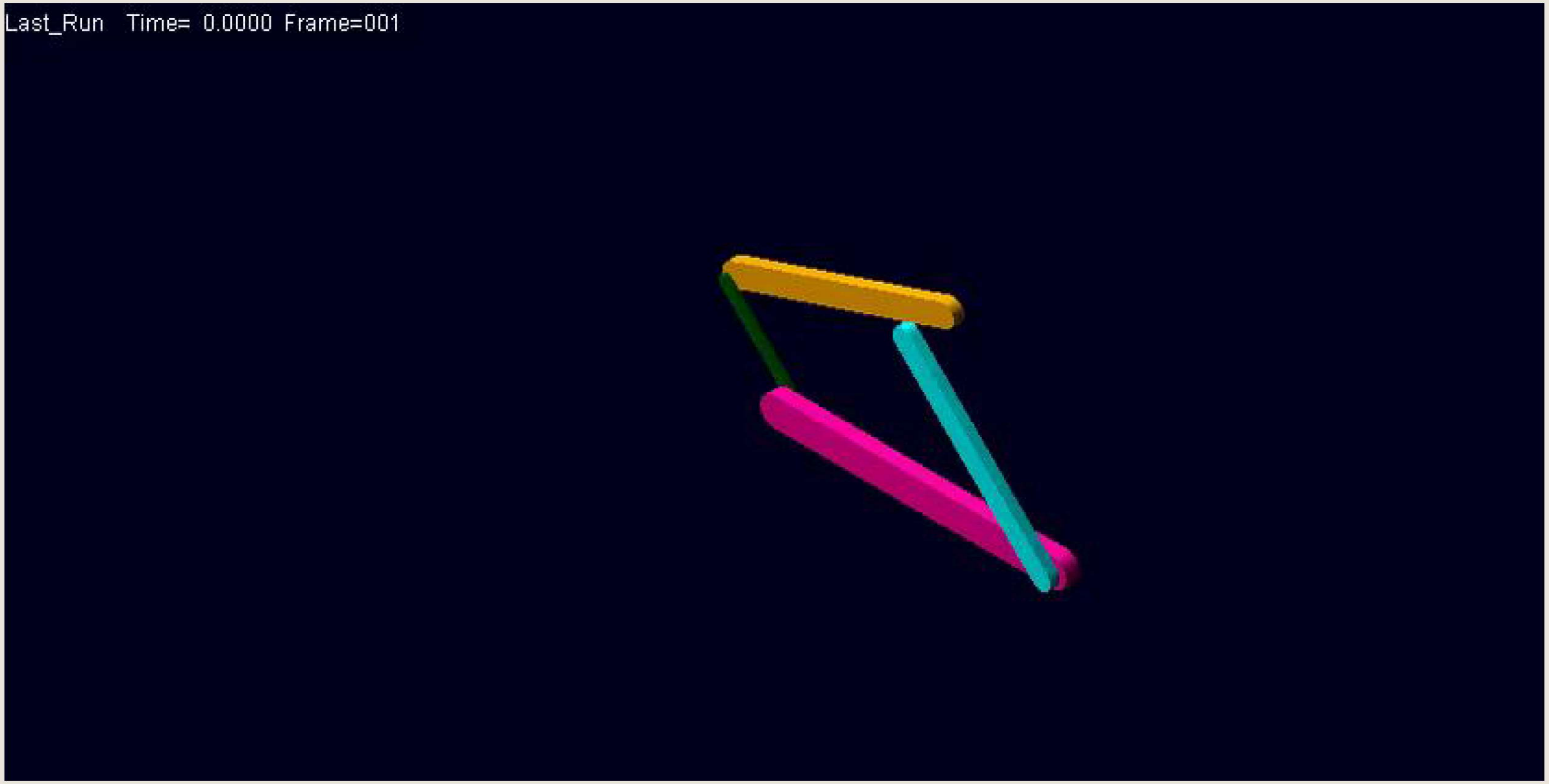
Crank – rocker
(a link attached to the shortest link is fixed)



Double rocker (coupler makes a full revolution)
(the link opposite to the shortest link is fixed)

Inversions of the Grashof Fourbar

$$S + L < P + Q$$



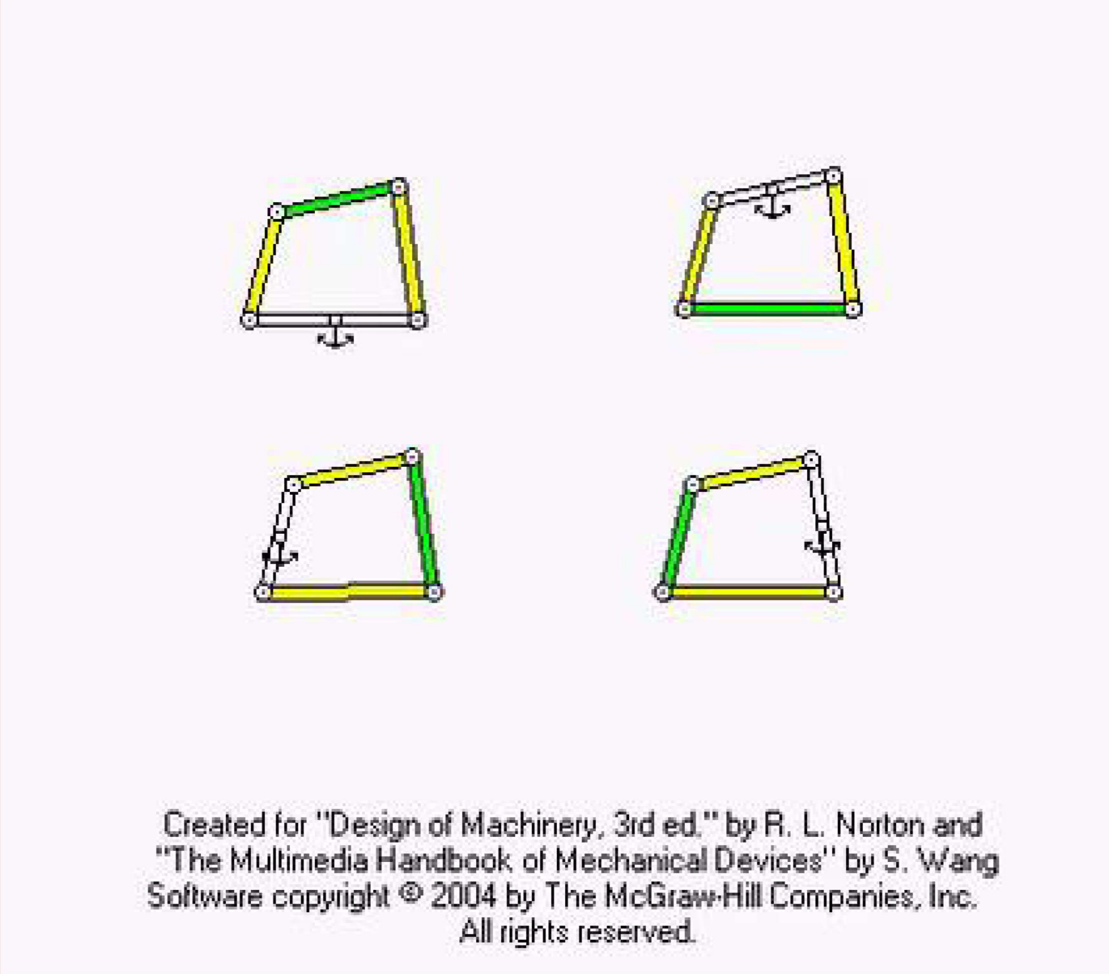
Double crank in isometric view

Note how the links move in different parallel planes without interference.

Inversions of the Non-Grashof Fourbar

*For the **Class II** case,* $S + L > P + Q$

- All inversions will be triple-rockers in which no link can fully rotate.



Inversions of the Non-Grashof Fourbar

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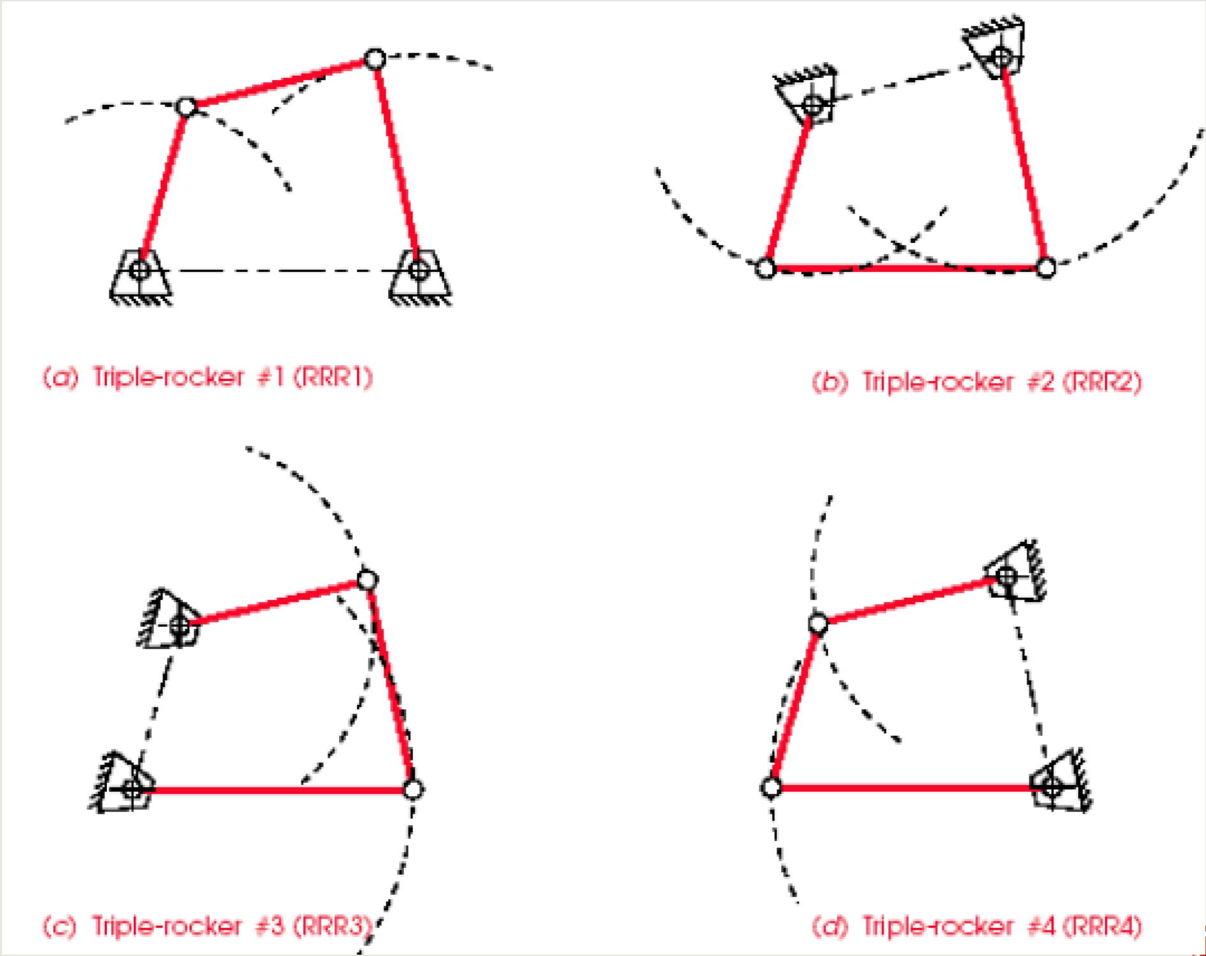


Triple-rockers

Special-Case Grashof Fourbar

$$S + L = P + Q$$

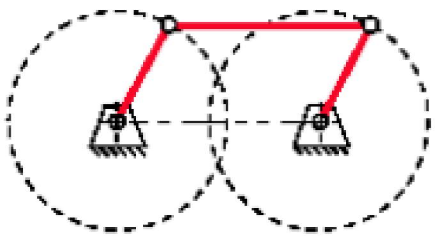
- Referred to as special-case Grashof and also as a **Class III** kinematic chain.

- All inversions will be either double-cranks or crank-rockers, but will have "change points" twice per revolution of the input crank when the links all become colinear.

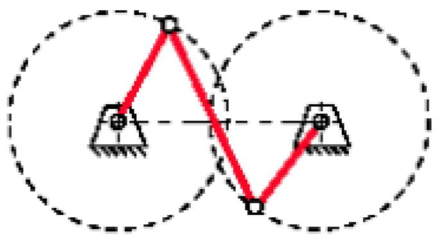
- At these change points the, output behavior will become indeterminate.

Special-Case Grashof Fourbar

$$S + L = P + Q$$

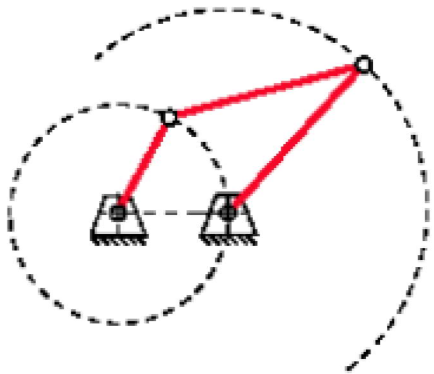


(a) Parallelogram form



(b) Antiparallelogram form

Double-crank or crank-rocker



(d) Deltoid or kite form

THANKS