Digital Signal Processing

1. ***Check whether the signal is Periodic or Aperiodic:***

x(t) = 2cos(10t+1) – sin(4t-1)

Time period $T_1 = \frac{2\pi}{\Omega_{01}} = \frac{2\pi}{10} = \frac{\pi}{5}$ *sec*

Time Period $T_2 = \frac{2\pi}{\Omega_{02}} = \frac{2\pi}{4} = \frac{\pi}{2}$ *sec*

$T = \frac{T_1}{T_2} = \frac{\pi/5}{\pi/2} = \frac{2}{5}$ *sec*

$T = 5T_1 = 2T_2$

$T = \frac{5\pi}{5} = \frac{2\pi}{2}$

$T = \pi$ *sec*

x(t) = cos 60πt + sin 50πt

$T_1 = \frac{2\pi}{60\pi} = \frac{1}{30}$ *sec*

$T_2 = \frac{2\pi}{50\pi} = \frac{1}{25}$ *sec*

$T = \frac{T_1}{T_2} = \frac{1/30}{1/25} = \frac{5}{6}$

$T = 6T_1 = 5T_2$

$T = \frac{1}{5}$ *sec*

x(t) = 3cos 4t + 2sin πt

$T_1 = \frac{2\pi}{4} = \frac{\pi}{2}$ *sec*

$T_2 = \frac{2\pi}{\pi} = 2$ *sec*

$T = \frac{T_1}{T_2} = \frac{\pi}{4}$ *this is not a rational number. So the signal is not periodic.*

x(n) = cos2πn

$N = \frac{2\pi}{\omega_0} = \frac{2\pi m}{2\pi}$ *(Put some small value of m so that N becomes an integer)*

$N = 1$

$\boldsymbol{x(n) = e^{j6\pi n}}$

$N = \frac{2\pi m}{\omega_0} = \frac{2\pi m}{6\pi} = \frac{3}{3}$

$N = 1$

$\boldsymbol{x(n) = e^{j\left(\frac{2\pi}{3}\right)n} + e^{j\left(\frac{3\pi}{4}\right)n}}$

$N_1 = \frac{2\pi m}{2\pi/3} = 3m$ for $m = 1$; $N_1 = 3$

$N_2 = \frac{2\pi m}{3\pi/4} = \frac{8m}{3}$ for $m = 3$; $N_2 = 8$

$\frac{N_1}{N_2} = \frac{3}{8} = N$

$N = 8N_1 = 3N_2$

$8(3) = 3(8) = 24$

$N = 24$

$\boldsymbol{x(n) = 12\cos(20n)}$

$N = \frac{2\pi m}{20} = \frac{\pi m}{10}$

For any values of m N is not an integer. So the given signal is aperiodic.

2. ***Check whether the systems are Time variant/invariant:***

a. $T[x(n)] = g(n)x(n)$

$y(n) = g(n)\,x(n)$

Shift the input by k

$y_1(n) = g(n).x(n-k) \longrightarrow 1$

Shift the output by k

$y(n-k) = g(n-k).x(n-k) \longrightarrow 2$

Shift in input and shift in output is not equal. So the system is time/shift variant.

b. $T[x(n)] = \sum_{k=n_0}^{n} x(k)$

$y(n) = \sum_{k=n_0}^{n} x(k)$

$y(n) = x(n_0) + x(n_0+1) + \ldots\ldots + x(n-1) + x(n)$

Shift the output by k

$y(n-k) = x(n_0) + x(n_0+1) + \ldots\ldots + x(n-k-1) + x(n-k) \longrightarrow 1$

Shift the input by k

$y_1(n) = x(n_0) + x(n_0+1) + \ldots\ldots + x(n-k-1) + x(n-k) \longrightarrow 2$

Shift in input and output do not vary. So the system is time invariant.

c. $T[x(n)] = e^{x(n)}$

$y(n) = e^{x(n)}$

Shift the input by k

$y(n) = e^{(n-k)} \longrightarrow 1$

Shift the output by k

$y(n-k) = e^{x(n-k)} \longrightarrow 2$

Shift the input and output do not vary. So the system is time invariant.

d. $y(n) = x(n)\cos\omega_0 n$

Shift the input by k

$y(n) = x(n-k)\cos\omega_0 n \longrightarrow 1$

Shift the output by k

$y(n-k) = x(n-k)\cos\omega_0(n-k) \longrightarrow 2$

Shift in input and output varies. So the system is time variant.

e. $y(n) = x(\frac{1}{2n})$

Shift the input by k

$y1(n) = x(\frac{1}{2(n-k)}) \longrightarrow 1$

Shift the output by k

$y(n-k) = x(\frac{1}{2(n-k)}) \longrightarrow 2$

The shift in input and output do not vary. So the system is time invariant.

f. $y(n) = x(n) + nx(n+1)$

Shift the input by k

$y1(n) = x(n-k) + n\,x(n-k+1) \longrightarrow 1$

Shift the output by k

y (n-k) = x (n-k) + (n-k) x (n-k+1) ⟶2

Shift in input and output varies. So the system is time variant

g. y(n) = sin x (n)

Shift the input by k

y1 (n) = sin x (n-k) ⟶1

Shift the output by k

y (n-k) = sin x (n-k) ⟶2

Shift in the input and output do not vary. So the system is time invariant.

3. a) Find the Z transform for the signal x (n) = $r^n \frac{\sin[(n+1)\omega]}{\sin\omega}$ ***u (n); 0<r<1***

__Solution:__

$X(Z) = \sum_{n=-\infty}^{\infty} x(n) Z^{-n}$

$X(Z) = \sum_{n=-\infty}^{\infty} r^n \frac{\sin[(n+1)\omega]}{\sin\omega} \; u(n) Z^{-n}$

$= \sum_{n=0}^{\infty} r^n \frac{\sin[(n+1)\omega]}{\sin\omega} \; Z^{-n} = \sum_{n=0}^{\infty} \frac{r^n}{\sin\omega} \frac{e^{j(n+1)\omega} - e^{-j(n+1)\omega}}{2j} \; Z^{-n}$

$= \sum_{n=0}^{\infty} \frac{r^n}{\sin\omega} \frac{e^{j(n+1)\omega}}{2j} \; Z^{-n} - \sum_{n=0}^{\infty} \frac{r^n}{\sin\omega} \frac{e^{-j(n+1)\omega}}{2j} \; Z^{-n}$

$= \sum_{n=0}^{\infty} \frac{r^n}{\sin\omega} \frac{e^{jn\omega}e^{j\omega}}{2j} \; Z^{-n} - \sum_{n=0}^{\infty} \frac{r^n}{\sin\omega} \frac{e^{-jn\omega}e^{-j\omega}}{2j} \; Z^{-n}$

$= \sum_{n=0}^{\infty} \frac{e^{j\omega}}{\sin\omega} \frac{e^{jn\omega}r^n}{2j} \; Z^{-n} - \sum_{n=0}^{\infty} \frac{e^{-j\omega}}{\sin\omega} \frac{e^{-jn\omega}r^n}{2j} \; Z^{-n}$

$= \sum_{n=0}^{\infty} \frac{e^{j\omega}}{\sin\omega} \left(\frac{e^{j\omega}r}{2j} \; Z^{-1}\right)^n - \sum_{n=0}^{\infty} \frac{e^{-j\omega}}{\sin\omega} \left(\frac{e^{-j\omega}r}{2j} \; Z^{-1}\right)^n$

$= \frac{e^{j\omega}}{2j\sin\omega} \frac{1}{1-e^{j\omega}rZ^{-1}} - \frac{e^{-j\omega}}{2j\sin\omega} \frac{1}{1-e^{-j\omega}rZ^{-1}} = \frac{1}{2j\sin\omega} \left[\frac{e^{j\omega}(1-e^{-j\omega}rZ^{-1}) - e^{-j\omega}(1-e^{j\omega}rZ^{-1})}{(1-e^{j\omega}rZ^{-1})(1-e^{-j\omega}rZ^{-1})}\right]$

$= \frac{1}{2j\sin\omega} \left[\frac{e^{j\omega} - rZ^{-1} - e^{-j\omega} + rZ^{-1}}{1-e^{-j\omega}rZ^{-1} - e^{j\omega}rZ^{-1} + r^2Z^{-2}}\right] = \frac{1}{2j\sin\omega} \left[\frac{2j\sin\omega}{1-rZ^{-1}\,(e^{-j\omega}+e^{j\omega}) + r^2Z^{-2}}\right]$

$X(Z) = \frac{1}{1-rZ^{-1}2\cos\omega + r^2Z^{-2}} = \frac{1}{Z^{-2}\,(Z^2 - rZ2\cos\omega + r^2)}$

$$X(Z) = \frac{Z^2}{(Z^2 - rZ2\cos\omega + r^2)}$$

3. b) Find the inverse Z transform of $X(Z) = \frac{Z+0.2}{(Z+0.2)(Z-1)}$, $|Z|>1$

Solution:

$$X(Z) = \frac{Z+0.2}{Z^2 - 0,5Z - 0.5}$$

$$\begin{array}{r|l}
 & Z^{-1} + 0.7Z^{-2} + 0.85Z^{-3} + 0.775Z^{-4} \\
\hline
Z^2 - 0.5Z - 0.5 & Z + 0.2 \\
 & Z - 0.5 - 0.5Z^{-1} \\
 & (-)\ (+)\ (+) \\
\cline{2-2}
 & 0.7 + 0.5Z^{-1} \\
 & 0.7 - 0.35Z^{-1} - 0.35Z^{-2} \\
 & (-)\ (+)\ (+) \\
\cline{2-2}
 & 0.85Z^{-1} + 0.35Z^{-2} \\
 & 0.85Z^{-1} - 0.425Z^{-2} - 0.425Z^{-3} \\
 & (-)\ (+)\ (+) \\
\cline{2-2}
 & 0.775Z^{-2} + 0.425Z^{-3} \\
 & 0.775Z^{-2} - 0.387Z^{-3}
\end{array}$$

$X(Z) = Z^{-1} + 0.7Z^{-2} + 0.85Z^{-3} + 0.775Z^{-4} + \ldots\ldots$

$= \sum_{n=0}^{\infty} x(n)\, Z^{-n}$

$X(0) = 0$, $x(1) = 1$, $x(2) = 0.7$, $x(3) = 0.85$, $x(4) = 0.775$, and so on

4. a) Find the 8 point DFT FFT of the sequence x(n)={1,2,3,4}.

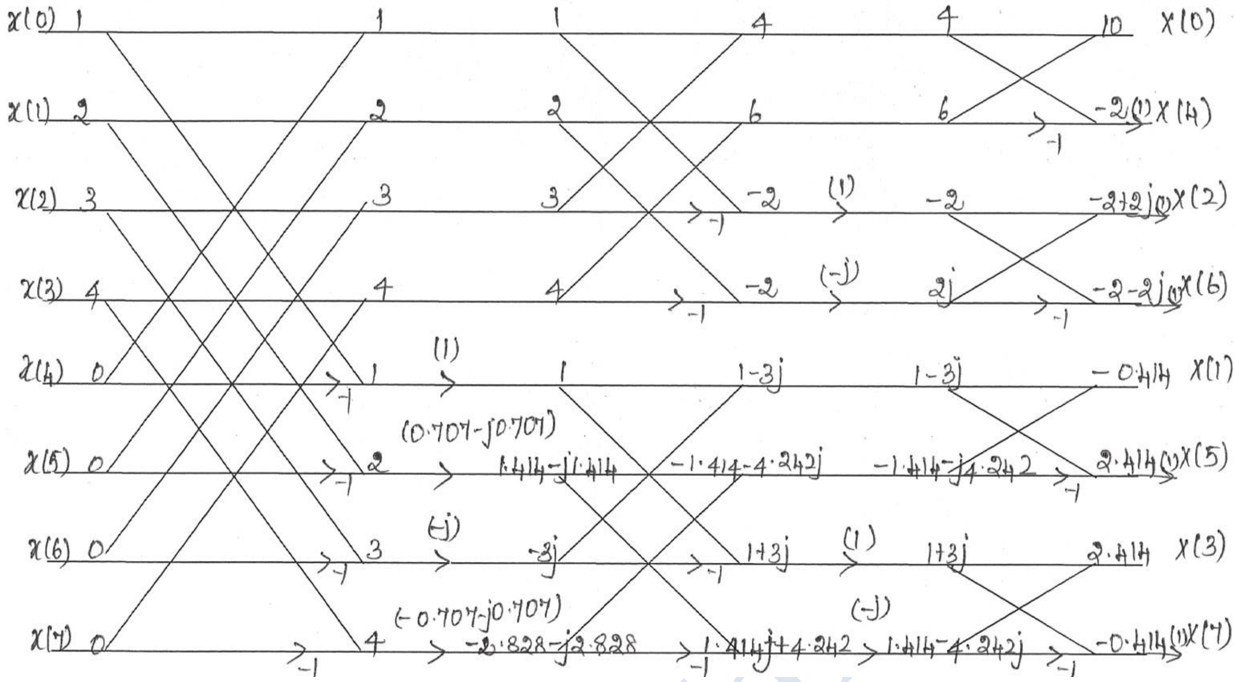
Solution

$W_8^0 = (e^{-j2\pi/8})^0 = 1$

$W_8^1 = (e^{-j2\pi/8})^1 = \cos\frac{\pi}{4} - j\sin\frac{\pi}{4} = 0.707 - j0.707$

$W_8^2 = (e^{-j2\pi/8})^2 = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} = 0 - j(1) = -j$

$W_8^3 = (e^{-j2\pi/8})^3 = \cos\frac{3\pi}{4} - j\sin\frac{3\pi}{4} = -\ 0.707 - j0.707$



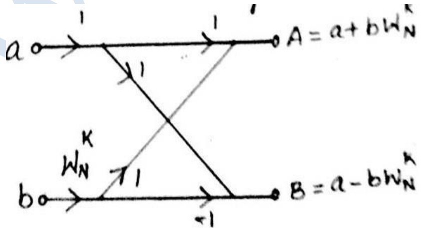
X(k)={10, -2, -2+2j, -2-2j, -0.414, 2.414, 2.414, -0.144}

4. b) ***Explain in detail the signal flow graph of DIT radix 2 FFT***

The basic computation involves

- *Two complex numbers a and b in each computation*
- *Complex number b is multiplied by a phase factor* W_n^k
- *The product* bW_n^k *is added to the complex number a to form new complex number A*
- *The product* bW_n^k *is subtracted from the complex number a to form new complex number B*

The basic butterfly or flow gram of DIT radix 2 FFT



In radix 2 FFT, N/2 butterflies per stage are required to represent the computational process

First stage of computation

$v_{11}(0)=x(0)+W_{N/4}^{0}x(4)$

$v_{11}(1)=x(0)-W_{N/4}^{0}x(4)$

$v_{12}(0)=x(2)+W_{N/4}^{0}x(6)$

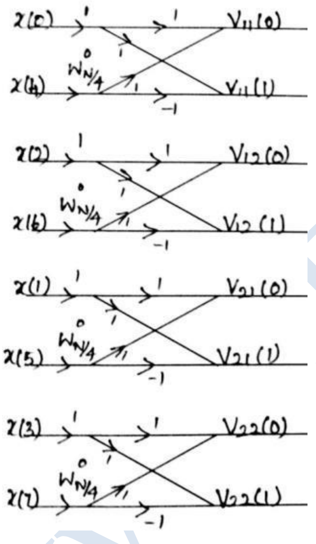
$v_{12}(1)=x(2)-W_{N/4}^{0}x(6)$

$v_{21}(0)=x(1)+W_{N/4}^{0}x(5)$

$v_{21}(1)=x(1)-W_{N/4}^{0}x(5)$

$v_{22}(0)=x(3)+W_{N/4}^{0}x(7)$

$v_{22}(1)=x(3)-W_{N/4}^{0}x(7)$



Second stage of computation

$F_1(0)= v_{11}(0)+ W_{N/4}^{0} v_{12}(0)$

$F_1(1)= v_{11}(1)+ W_{N/4}^{0} v_{12}(1)$

$F_1(2)= v_{11}(0)- W_{N/4}^{0} v_{12}(0)$

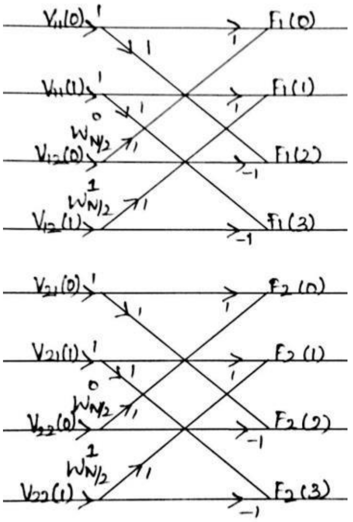
$F_1(3)= v_{11}(1)- W_{N/4}^{0} v_{12}(1)$

$F_2(0) = v_{21}(0) + W^0_{N/4} v_{22}(0)$

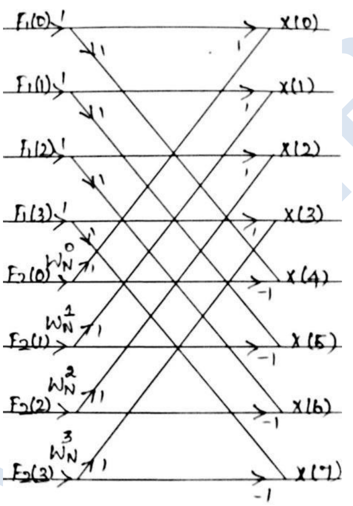
$F_2(1) = v_{21}(1) + W^0_{N/4} v_{22}(1)$

$F_2(2) = v_{21}(0) - W^0_{N/4} v_{22}(0)$

$F_2(3) = v_{21}(1) - W^0_{N/4} v_{22}(1)$



Third stage of computation



$X(0) = F_1(0) + W^0_N F_2(0)$

$X(1) = F_1(1) + W^1_N F_2(1)$

$X(2) = F_1(2) + W^2_N F_2(2)$

$X(3) = F_1(3) + W^3_N F_2(3)$

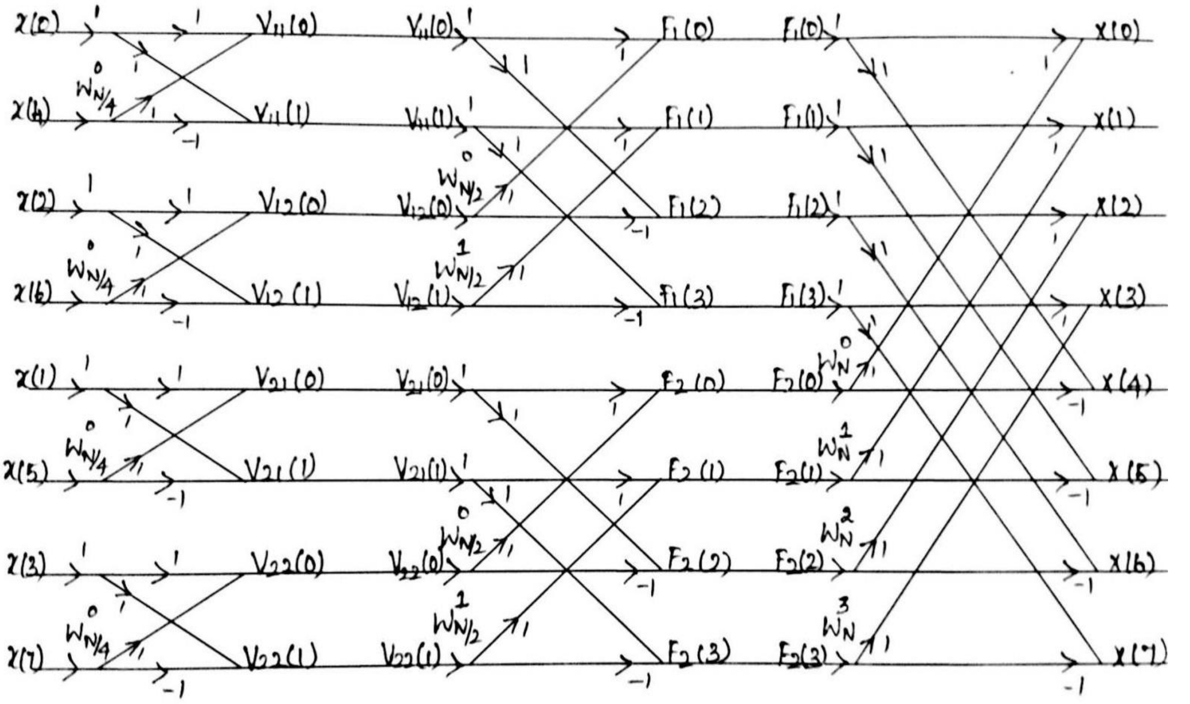
$X(4) = F_1(4) + W_N^4 F_2(4)$

$X(5) = F_1(5) + W_N^5 F_2(5)$

$X(6) = F_1(6) + W_N^0 F_2(6)$

$X(7) = F_1(7) + W_N^0 F_2(7)$

Combined butterfly diagram is



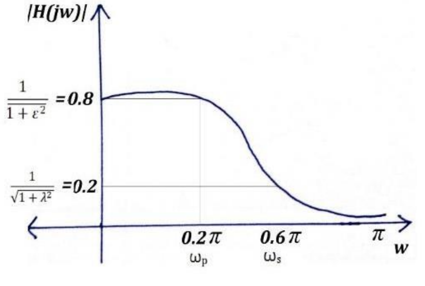
5. Design a butter worth filter using bilinear transformation technique

$$0.8 \le |H(e^{j\omega})| \le 1 \quad ; 0 \le \omega p \le 0.2\pi$$

$$|H(e^{j\omega})| \le 0. \ ; 0.6\pi \le \omega s \le \pi$$

<u>Solution:</u>

Step 1: draw the filter characteristics



Step 2: collect the given parameters $\omega_p = 0.2$, $\omega_s = 0.6\pi$

$$\frac{1}{\sqrt{1+\varepsilon^2}} = 0.8 \Rightarrow 0.8\sqrt{1+\varepsilon^2} = 1 \Rightarrow \frac{1}{0.8} = \sqrt{1+\varepsilon^2} \Rightarrow 1+\varepsilon^2 = \left(\frac{1}{0.8}\right)^2 \Rightarrow$$

$\varepsilon = 0.75$

$$\frac{1}{\sqrt{1+\lambda^2}} = 0.2 \Rightarrow \lambda = 4.8989$$

Step 3: Select the transformation technique (BLTT)

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2} \Rightarrow \frac{2}{1}\tan\frac{0.2\pi}{2} = 0.6498$$

$$\Omega s = \frac{2}{T}\tan\frac{\omega_s}{2} \Rightarrow \frac{2}{1}\tan\frac{0.6\pi}{2} = 2.7527$$

Step 4: Find the order of filter

$$\text{For LPF, } N \geq \frac{lo(\lambda/\varepsilon)}{lo(\Omega_s/\Omega_p)} \Rightarrow \frac{lo(4.8989/0.75)}{lo(2.7527/0.6498)}$$

On simplifying

$N \geq 1.299 \Rightarrow N = 2$

Step 5: Butterworth polynomial for order 2, $S^2 + \sqrt{2}S + 1$

Step 6: Transfer function of normalized LPF for N=2 (by substituting the Butterworth polynomial for order 2)

$$|H(j\omega)| = \frac{1}{S^2+\sqrt{2}S+1}$$

Step 7: Denormalize the transfer function

For LPF, $S \rightarrow \frac{S}{\Omega_c}$

$$\Omega c = \frac{\Omega_P}{\varepsilon^{1/N}} \Rightarrow \frac{0.6498}{(0.75)^{1/2}} = 0.7503$$

$$S \rightarrow \frac{S}{0.7503}$$

$$H(S) = \frac{1}{\left[\frac{S}{0.7503}\right]^2 + \sqrt{2}\left[\frac{S}{0.7503}\right] + 1}$$

$$= \frac{0.5629}{S^2+\sqrt{2}(0.7503)+0.5629} = \frac{0.5629}{S^2+1.061S+0.5629}$$

Step 8: convert analog filter into respective digital filter using BLTT

Replace $S \rightarrow \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$, $T=1$

$$H(z) = \frac{0.5629}{2^2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 1.061(2)\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+0.5629}$$

$$= \frac{0.5629\,(1+z^{-1})^2}{4(1-z^{-1})^2+2.122(1-z^{-1})(1+z^{-1})+0.5629(1+z^{-1})^2}$$

$$= \frac{0.5629\,(1+2z^{-1}+z^{-2})}{4(1-2z^{-1}+z^{-2})+2.122(1-z^{-2})+0.5629(1+2z^{-1}+z^{-2})}$$

$$H(Z) = \frac{0.5629+1.1258z^{-1}+0.5629z^{-2}}{6.6849-6.8742z^{-1}+2.4409z^{-2}}$$

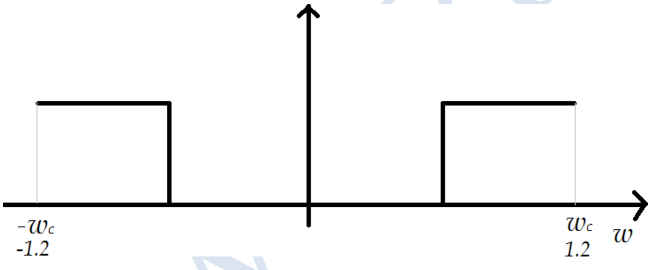
$$H(Z) = \frac{0.5629+1.1258z^{-1}+0.5629z^{-2}}{6.6849[1-1.0283z^{-1}+0.3651z^{-2}]}$$

Step 9: final transfer function

$$H(Z) = \frac{0.0842+0.1684z^{-1}+0.0842z^{-2}}{1-1.0283z^{-1}+0.3651z^{-2}}$$

6. Design a HPF, using hamming window with a cut off frequency 1.2rad/sec and N=9.

Solution:



Given: $N=9$ $\omega_c=1.2$ *rad/sec*

For HPF desired frequency response is

$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha}; & -\pi \le \omega \le -\omega_c \text{ and } \omega_c \le \omega \le \pi \\ 0; & otherwise \end{cases}$$

$$\alpha = \frac{N-1}{2} \Rightarrow \alpha = \frac{9-1}{2} \Rightarrow \alpha = 4$$

$$H_d(\omega) = \begin{cases} e^{-j\omega 4}; & -\pi \le \omega \le -1.2 \text{ and } 1.2 \le \omega \le \pi \\ 0; & otherwise \end{cases}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)e^{j\omega n}d\omega \quad [H_d(e^{j\omega}) = H_d(\omega)]$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{-1.2} e^{-j\omega 4} e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{1.2} e^{-j\omega 4} e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-1.2} e^{j\omega(n-4)} d\omega + \frac{1}{2\pi}\int_{1.2}^{\pi} e^{j(n-4)} d\omega$$

$$= \frac{1}{2\pi}\left\{\frac{e^{j(n-4)}}{(n-4)}\right\} + \frac{1}{2\pi}\left\{\frac{e^{j(n-4)}}{(n-4)}\right\}$$

$$= \frac{1}{2\pi(n-4)}\{e^{-1.2(n-4)} - e^{-\pi j(n-4)}\} + \frac{1}{2\pi(n-4)}\{e^{1.2(n-4)} - e^{\pi j(n-4)}\}$$

$$= \frac{1}{(n-4)}\left\{\left\{\frac{e^{-j1.2(n-4)} - e^{j1.2(n-4)}}{2j}\right\} + \left\{\frac{e^{j\pi(n-4)} - e^{-j\pi(n-4)}}{2j}\right\}\right\}$$

$$= \frac{1}{\pi 2(n-4)}\{e^{-j1.2(n-4)} - e^{j1.2(n-4)} + e^{j\pi(n-4)} - e^{-j\pi(n-4)}\}$$

$$h_d(n) = \frac{1}{(n-4)}\{in[\pi(n-4)] - sin[1.2(n-4)]\}$$

Now, n=0 to N-1 and here n= 0 to 8

$$h_d(0) = \frac{1}{(0-4)}\{sin(0-4) - sin1.2(0-4)\}$$

$$h_d(0) = \frac{1}{-12.5663}(0-0.9961) = 0.0792$$

$$h_d(1) = \frac{1}{(1-4)}\{sin\,\pi\,(1-4) - sin1.2\,(1-4)\}$$

$$h_d(1) = \frac{1}{-9.4247}(0-0.4425) = 0.0478$$

$$h_d(2) = \frac{1}{(2-4)}\{sin\,\pi\,(2-4) - sin1.2\,(2-4)\}$$

$$h_d(2) = \frac{1}{-6.2831}(0+0.6754) = -0.1075$$

$$h_d(3) = \frac{1}{(3-4)}\{sin\,\pi\,(3-4) - sin1.2\,(3-4)\}$$

$$h_d(3) = \frac{1}{-3.1415}(0+0.9320) = -0.2966$$

$$h_d(4) = \frac{1}{(4-4)} sin\,\pi\,(4-4) - sin1.2\,(4-4)\} = \infty \text{ (infinity)}$$

So applying L'hospital rule,

At n=α=4

$$h_d(4) = \frac{1}{(n-4)}\{sin\pi\,(n-4) - sin1.2\,(n-4)\}$$

$$= \frac{sin(n-4).1}{(n-4)}\ \frac{1}{\pi}\frac{sin\,1.2(n-4)}{(n-4)}$$

$h_d(4) = 1 - \frac{1}{\pi} = \frac{\pi - 1.2}{\pi} = 0.6180$

$h_d(5) = \frac{1}{(5-4)}\{\sin \pi(5-4) - \sin 1.2(5-4)\}$

$h_d(5) = \frac{1}{\pi}(0 - 0.9320) = -0.2966$

$h_d(6) = \frac{1}{(6-4)}\{\sin \pi(6-4) - \sin 1.2(6-4)\}$

$h_d(6) = \frac{1}{2\pi}(0 - 0.6754) = -0.1075$

$h_d(7) = \frac{1}{(7-4)}\{\sin \pi(7-4) - \sin 1.2(7-4)\}$

$h_d(7) = \frac{1}{3\pi}(0 - 0.4425) = 0.0478$

$h_d(8) = \frac{1}{(8-4)}\sin \pi(8-4) - \sin 1.2(8-4)\}$

$h_d(8) = \frac{1}{4\pi}(0 + 0.9961) = 0.0792$

Now, to find $\omega_H(n)$

$$\omega_H(n) = \begin{cases} 0.54 - 0.46\cos\frac{2n}{N}; & 0 \le n \le N-1 \\ 0; & otherwise \end{cases}$$

$\omega_H(0) = 0.54 - 0.46\cos(0) = 0.08$

$\omega_H(1) = 0.54 - 0.46\cos\left(\frac{\pi}{4}\right) = 0.2147$

$\omega_H(2) = 0.54 - 0.46\cos\left(\frac{\pi}{2}\right) = 0.54$

$\omega_H(3) = 0.54 - 0.46\cos\left(\frac{\pi}{8}\right) = 0.8652$

$\omega_H(4) = 0.54 - 0.46\cos(\pi) = 1$

$\omega_H(5) = 0.54 - 0.46\cos\left(\frac{10\pi}{8}\right) = 0.8652$

$\omega_H(6) = 0.54 - 0.46\cos\left(\frac{12\pi}{8}\right) = 0.54$

$\omega_H(7) = 0.54 - 0.46\cos\left(\frac{14\pi}{8}\right) = 0.2147$

$\omega_H(8) = 0.54 - 0.46\cos(2\pi) = 0.08$

$h(n) = h_d(n)\,\omega_H(n)$

$h(0) = h_d(0)\,\omega_H(0) = 0.0792 \times 0.08 = 6.335 \times 10^{-3}$

$h(1) = h_d(1)\,\omega_H(1) = 0.0478 \times 0.2147 = 0.01006$

$h(2) = h_d(2)\,\omega_H(2) = -0.1075 \times 0.54 = =0.05799$

$h(3) = h_d(3)\,\omega_H(3) = -0.2966 \times 0.8652 = -0.2566$

$h(4) = h_d(4)\,\omega_H(4) = 0.6180 \times 1 = 0.6180$

$h(5) = h_d(5)\,\omega_H(5) = -0.2966 \times 0.8652 = -0.2566$

$h(6) = h_d(6)\,\omega_H(6) = -0.1075 \times 0.54 = =0.05799$

$h(7) = h_d(7)\,\omega_H(7) = 0.0478 \times 0.2147 = 0.01006$

$h(8) = h_d(8)\, \omega_H(8) = 0.0792 \times 0.08 = 6.335\times10^{-3}$

Magnitude response of N= odd,

$$|H(w)| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right)\cos \omega n$$

$$= h(4) + \sum_{n=1}^{4} 2\,h(4-n)\cos \omega n$$

$$= h(4) + [2h(3)\cos\omega + 2h(2)\cos 2\omega + 2h(1)\cos 3\omega + 2h(0)\cos 4\omega]$$

$$= 0.6180 + \{2(-0.2566)\cos\omega + 2(-0.05799)\cos 2\omega + 2(0.01006)\cos 3\omega + 2(6.335\times10^{-3})\cos 4\omega\}$$

$$= 0.6180 + \{-0.5132\cos\omega - 0.11598\cos 2\omega + 0.02012\cos 3\omega + 0.12672\cos 4\omega\}$$

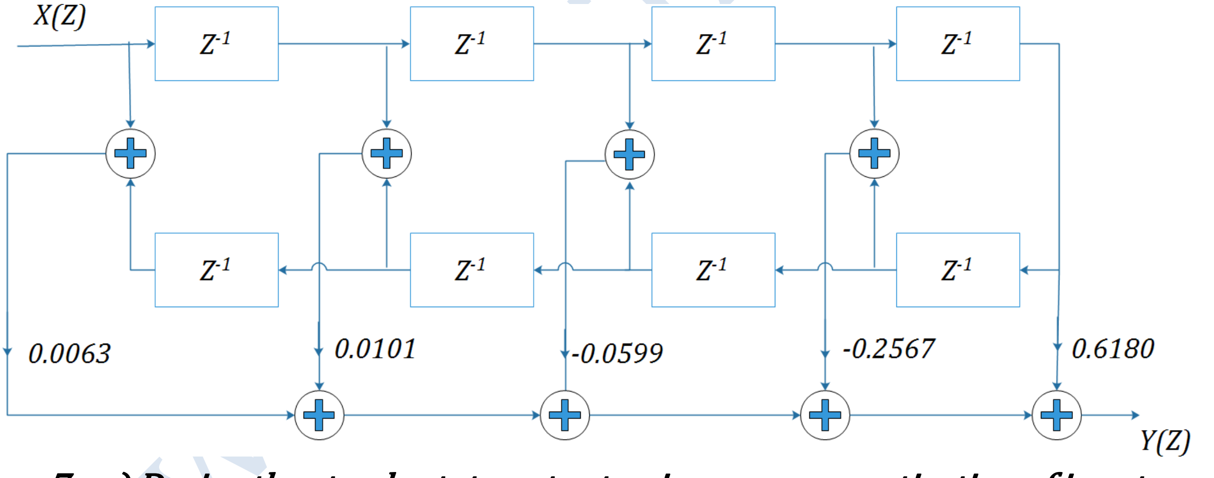
Transfer function,

$$H(z) = \sum_{n=0}^{N-1} h(n)^{-n}$$

$$= h(0)z^{0} + h(1)z^{1} + h(2)z^{2} + h(3)z^{3} + h(4)z^{4} + h(5)z^{5} + h(6)z^{6} + h(7)z^{7} + h(8)z^{8}$$

$$= 6.335\times10^{-3} z^{0} + 0.01006 z^{1} - 0.05799 z^{2} - 0.2566 z^{3} + 0.6180 z^{4} - 0.2566 z^{5} - 0.05799 z^{6} + 0.01006 z^{7} + 6.335\times10^{-3} z^{8}$$

$$= 6.335\times10^{-3}(z^{0}+z^{8}) + 0.0101(z^{1}+z^{7}) - 0.05799(z^{2}+z^{6}) - 0.2566(z^{3}+z^{5}) + 0.6180(z^{4})$$



7. a) Derive the steady state output noise power quantization of input data.

Quantization step size – q

$Q = \frac{R}{2^b}$ (for 2's complement)

$Q = \frac{R}{2^{b-1}}$ (for sign magnitude and 1's complement)

Let x (n) =Unquantised sample of the signal.

$x_q(n)$ =quantised sample of the signal.

$e(n)$ =quantization error.

$e(n) = x_q(n) - x(n)$

Range of $e(n) = \frac{-q}{2}$ to $\frac{q}{2}$

Mean value or expected value of error signal=$E\{e\}$

$$E\{e\} = \frac{1}{\frac{q}{2}-(-\frac{q}{2})} \int_{-q/2}^{q/2} e\, de$$

$$= \frac{1}{q}\left[\frac{e^2}{2}\right]_{\frac{-q}{2}}^{\frac{q}{2}}$$

$$= \frac{1}{2q}\left[\frac{^2}{4} - \frac{q^2}{4}\right]$$

$E\{e\} = 0$

Variance of error signal $\sigma_e^2 = E\{e^2\} - E^2\{e\}$

$E\{e\} = 0$ so $E^2\{e\} = 0$

$$E\{e\} = \frac{1}{\frac{q}{2}-(-\frac{q}{2})} \int_{-q/2}^{q/2} e^2 de$$

$$= \frac{1}{q}\left[\frac{e^3}{3}\right]_{-q/2}^{q/2}$$

$$= \frac{1}{3q}\left[\frac{^3}{8} - \frac{-q^3}{8}\right]$$

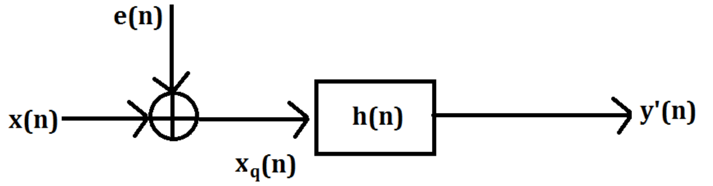
$$= \frac{1}{3q}\left[\frac{2q^3}{8}\right]$$

$\sigma_e^2 = \frac{q^2}{12}$ Where $q = \frac{R}{2^b}$

$\therefore \sigma_e^2 = \frac{R^2}{2^{2b}12} 12 = \frac{R^2 2^{-2b}}{12}$

When $R=2$, $\sigma_e^2 = \frac{2^{-2b}}{3}$

Steady state noise power due to the quantization error signal



h (n) ⟶*impulse response*

y' (n) ⟶*response*

$y'(n) = x_q(n) * h(n)$

$= [x(n) + e(n)] * h(n)$

$= [x(n) * h(n)] + [e(n) * h(n)]$

$= y(n) + (n)$

Where $y(n) = x(n) * y(n)$

$(n) = e(n) * h(n)$

(n) = *output noise power or steady state output noise power (variance) due to quantization error signal*

Steady state output noise power due to quantization error $= \sigma_{eo}^2\ ^2 = \sigma_e^2 \sum_{n=0}^{\infty} h^2(n)$

$= \sigma_e^2 \frac{1}{2\pi j} \oint_N (Z)\, H(Z^{-1})\, Z^{-1} dz$

$= \sigma_e^2 \sum_{i=1}^{N} re[H(Z)\, H(Z^{-1})\, Z^{-1}]\, |_{z=pi}$

$Pi = P_1, P_2, \ldots\ldots P_N.$

7. b) ***Explain the characteristics of a limit cycle oscillation with respect to the system described in the equation***
y (n) =0.95 y (n-1) +x (n). Determine the dead band.

Solution:

$y(n) = 0.95 y(n-1) + x(n)$

$y'(n) = Q[0.95 y'(n-1)] + x(n)$

Assume

$y'(n) = 0$ for $n<0$

$$x(n) = \begin{cases} 0.75 \; for \; n = 0 \\ 0 \; for \; n \neq 0 \end{cases}$$

When $n=0$

$y'(n) = Q[0.95y'(-1)] + x(0)$

$= Q[0.95x0] + 0.75$

$= 0.75$

$= 0.1100_2$

When $n=1$

$y'(n) = Q[0.95y'(1-1)] + x(1)$

$= Q[0.95x0.75] + 0
$

$= Q[0.7125]$

$$.7125 \xrightarrow{to\ binary} .10110_2 \xrightarrow{add\ sign\ bit} 0.10110_2 \xrightarrow{round\ off\ to\ 4\ bits}$$

$$0.6875 \xleftarrow{convert\ to\ decimal} +.1011_2 \xleftarrow{extract\ sign\ bit} 0.1011_2$$

$0.7125x2 = 1.425$

$0.425x2 = 0.85$

$0.85x2 = 1.7$

$0.7x2 = 1.4$

$0.4x2 = 0.8$

$.1011$

$1x2^{-4} = 0.0625$

$1x2^{-3} = 0.125$

$0x2^{-2} = 0$

$1x2^{-1} = 0.5$

$y'(1) = 0.6875$

When $n=2$

$y'(2) = Q[0.95y'(2-1)] + x(2)$

$= Q[0.95x0.6875] + 0$

$=Q\,[0.653125]$

$$+.653125 \xrightarrow{\text{to binary}} +.101001_2 \xrightarrow{\text{add sign bit}} 0.101001_2 \xrightarrow{\text{round to 4 bits}}$$

$$+.625 \xleftarrow{\text{to decimal}} +.1010 \xleftarrow{\text{extract sign bit}} 0.1010$$

$.653125 \times 2 = 1.30625$

$.30625 \times 2 = 0.6125$

$.6125 \times 2 = 1.225$

$.225 \times 2 = 0.45$

$.45 \times 2 = 0.9$

$.9 \times 2 = 1.8$

$= 0.101001_2$

$.1010$

$0 \times 2^{-4} = 0$

$1 \times 2^{-3} = {}^{1}/_{8}$

$0 \times 2^{-2} = 0$

$1 \times 2^{-1} = {}^{1}/_{2}$

$y'(2) = 0.625$

When $n=3$

$y'(3) = Q\,[0.95y'(3-1)] + x(3)$

$= Q\,[0.95 \times 0.625] + 0$

$= Q\,[0.59375]$

$$+.59375 \xrightarrow{\text{to binary}} +.10011_2 \xrightarrow{\text{add sign bit}} 0.10011_2 \xrightarrow{\text{round to 4 bits}}$$

$$+.625 \xleftarrow{\text{to decimal}} +.1010 \xleftarrow{\text{extract sign bit}} 0.1010$$

System enters the limit cycle when $n=2$

Dead band $= \pm \frac{2^{-b}}{1-|a|}$ here $b=5$ $\quad |a| = 0.95$

$= \pm \frac{2^{-5}}{1-0.95} = \pm\, 0.625 = [+0.625, -0.625]$

1. State the sampling theorem

The sampling frequency must be at least the highest frequency present in the signal

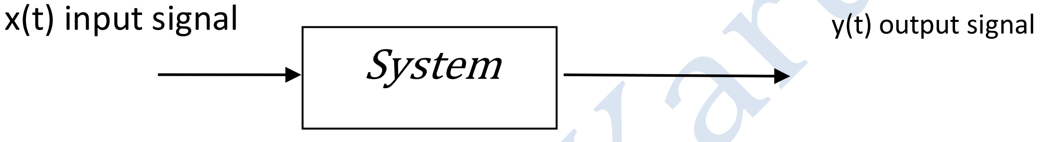
$F \geq 2f_m$

2. Define a signal

A signal is defined as any physical quantity that varies with time, space or any other independent variable.

3. define a system

A system is defined as an entity that manipulates one or more signals to accomplish a function, therefore producing new signal.



4. What is the condition for stability?

A system is said to be stable if it produces a bounded output for every bounded input (BIBO).

The system which does not satisfy this condition is an unstable system.

Condition for Stability:

$$\int_{-\infty}^{\infty} | h(t) | \, dt < \infty$$

5. State the superposition theorem

The response to a weighted sum of a signal be equal to the corresponding weighted sum of the outputs of the system to each of the individual input signal,

A system is said to be linear if and only if

$$T\,[a_1 x_1(n) + a_2 x_2(n)] = a_1 T\,[x_1(n)] + a_2 T\,[x_2(n)]$$

A system is said to be non-linear if it doesn't obey superposition principle.

6. Define Z transform.

Z transform converts difference equations into algebraic equations thereby simplifying the analysis of discrete time systems

Definition

The Z transform of a discrete time sequence x (n) is defined as

$X(Z) = \sum_{n=-\infty}^{\infty} x(n) Z^{-n}$

7. What is bilateral Z transform?

The Z transform of a discrete time sequence x (n) is defined as

$X(Z) = \sum_{n=-\infty}^{\infty} x(n) Z^{-n}$

If the sequence x (n) exists for n in the range -∞ to ∞ the above equation represents two sided or bilateral Z transform

8. What is unilateral Z transform?

If the sequence exists only for n≥0 then the equation changes to

$X(Z) = \sum_{n=0}^{\infty} x(n) Z^{-n}$

Which is called one sided Z transform or unilateral Z transform

9. Define inverse Z transform.

Inverse Z transform of X (z) is

$X(n) = \frac{1}{2\pi j} \oint_c X(Z) Z^{n-1} dZ$

10. Define Region of convergence.

ROC is the region where Z transform converges. From definition of Z transform it is clear that Z transform is an infinite power series

11. What is zero padding? Why it is needed?

Appending zeros to a sequence in order to increase the size or length of the sequence is called zero padding.

During convolution when two input sequences are of different size then they are converted to equal size by zero padding.

12. What is sectioned convolution?

In linear convolution of two sequences if one sequence is very much longer, the longer sequence is sectioned(splitted) into smaller sequence equal to the size of the smaller sequence and then the convolution is performed. The output sequences obtained are finally combined to get the overall output sequence this technique is called sectioned convolution

13. What is radix 2 FFT?

The radix 2 FFT is an efficient algorithm for computing N point DFT of a N point sequence. In radix 2 FFT the N point sequence is decimated into 2 point sequences and the 2 point DFT for each decimated sequence is computed. From the result of 2 point DFT the 4 point DFTs are computed. From the 4 point DFTs the 8 point DFTs are computed and so on until we get N point DFT

14. How many complex multiplications and additions are involved in DFT and FFT?

In FFT

Complex multiplications involved - $\frac{N}{2}\log_2 N$

Complex additions involved – $N \log_2 N$

In DFT

Complex multiplications involved – N^2

Complex additions involved – $N (N-1)$

15. What is decimation in time radix 2 FFT?

The DIT radix 2 FFT is an efficient algorithm for computing DFT. In DIT the time domain N point sequence is decimated into 2 point sequences. The result of 2 point DFTs are used to compute 4 point DFTs. The two numbers of 2 point DFTs are combined to get 4 point DFT. the result of 4 point DFTs are used to compute 8 point DFTs. Two numbers of 4 point DFTs are combined to get an 8 point DFT. This process is continued until we get N point DFT.

16. What is phase factor or twiddle factor?

The complex number W_N is called phase factor or twiddle factor. W_N represents a complex number $1 \angle -2\pi/N$ or $e^{-j2\pi/n}$. It also represents the Nth root of unity.

17. What are the basic elements used to construct the block diagram?

- *Adder*
- *Multiplier*
- *Delay unit*

18. List the types of structures for realizing IIR?

- *Direct form 1*
- *Direct form 2*
- *Cascade form*
- *Parallel form*

19. What is the advantage of Direct Form1 over Direct Form2?

In direct form 2 structure the number of delay elements requires is exactly half that of DF1, when the number of poles and zeros are equal. Hence it requires less memory.

20. What are the difficulties in cascade realization?

- *Decision of pairing poles and zeros.*
- *Deciding the order of cascading the first and second order sections.*
- *Scaling multipliers should be provided between individual sections to prevent the filter variables from becoming too large or too small.*

21. What is the advantage in cascade and parallel realization?

During digital implementation the filter coefficients are quantized. This may change the values of poles. This can be minimized by using cascade and parallel realization.

22. What is Gibb's phenomenon?

In FIR filter design by Fourier series method or rectangular window method, the infinite duration impulse response is truncated to finite duration impulse response. The abrupt truncation of impulse response introduces oscillations in the passband and stopband. This effect is called Gibb's oscillation.

23. What are the steps involved in FIR filter design?

1. *Choose the desired (ideal) frequency response Hd (w).*
2. *Take IFT of Hd (w) to get hd (n).*
3. *Convert the infinite duration hd (n) to finite duration sequence h (n).*
4. *Take z-transform of h (n) to get the transfer function H (z) of the FIR filter.*

1. *Multiply H(z) by $z^{-(N-1)/2}$ to convert the noncausal transfer function to a realizable causal FIR filter transfer function.*

24. Write the procedure for designing FIR filters using window technique.

1. *Choose the desired frequence response of the filter $H_d(w)$.*
2. *Take IFT of $H_d(w)$ to obtain the desired impulse response $h_d(n)$.*
3. *Choose a window sequence w(n) and multiply $h_d(n)$ with w(n) to convert the infinite duration impulse response to finite duration impulse response h(n).*

4. Take z-transform of h(n) to find the transfer function H(z) of the filter.

(i) The mean value of rounding error signal is zero
(ii) The variance of the rounding error signal is least

25. What is limit cycle?

In recursive systems, when the input is zero or some non-zero constant value, the non linearities due to finite precision arithmetic operation may cause periodic oscillations in output. During periodic oscillations the output will oscillate between a finite positive and negative value the output becomes constant for increasing n. Such oscillation is called limit cycles.

26. Define dead band

In a limit cycle the amplitudes of the output are confined to a range of values called dead band of the filter

$$\text{Dead band} = \pm\frac{2^{-b}}{1-|\alpha|} = \left[\frac{-2^{-b}}{1-|\alpha|}, \frac{+2^{-b}}{1-|\alpha|}\right]$$

b- Number of bits (including sign bit)