Ch. (1) - Kinematics of a Rigid Body

Type of Motion for Rigid Body

Translation | Rotation about fixed axis | General Plane Motion (G.P.M)

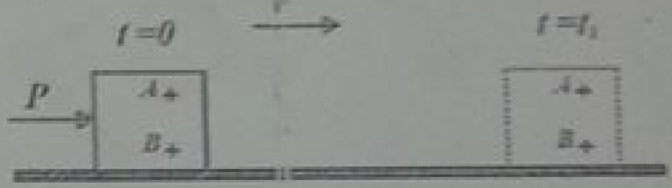
1-Translation: حركه انتقاليه

البعد بين اى نقطتين على الجسم ثابت ولا يتغير مع الحركه

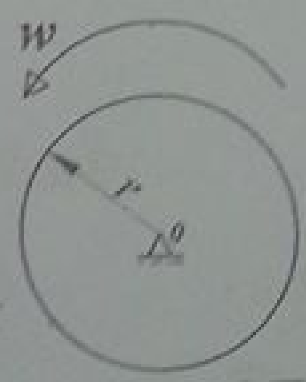
$\therefore S_A = S_B$

$\therefore v_A = v_B$

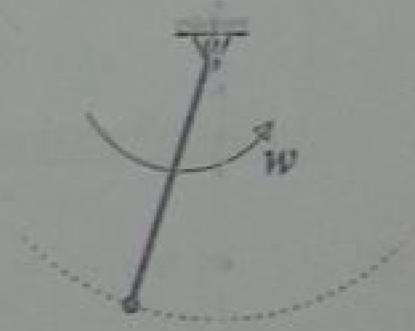
$\therefore a_A = a_B$



2- Rotation about fixed axis: حركه دورانيه حول محور ثابت



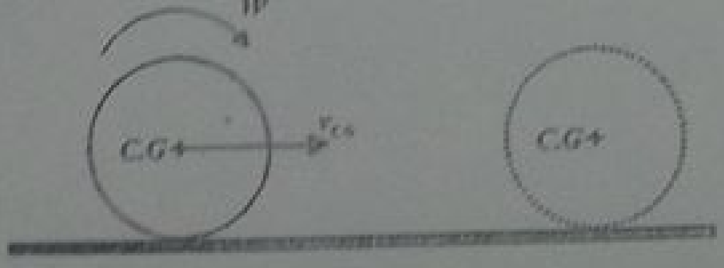
Gear or Pulley or Disk



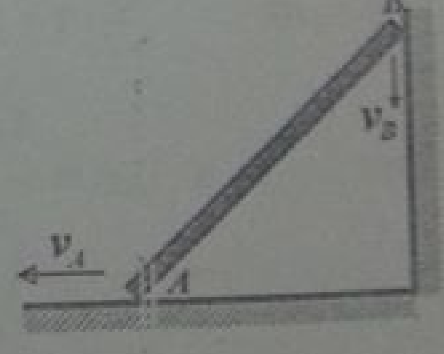
Pendulum

3- General Plane Motion (G.P.M): حركه عامه داخل المستوى

G.P.M = Translation + Rotation

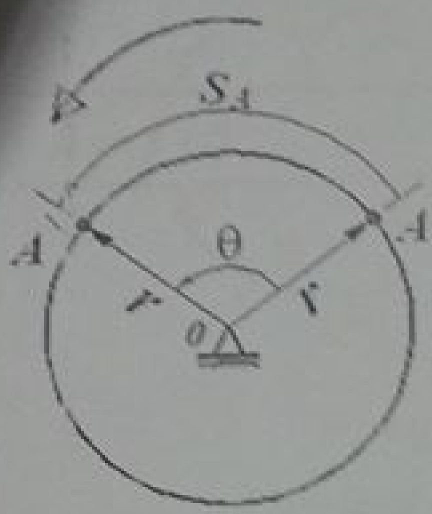


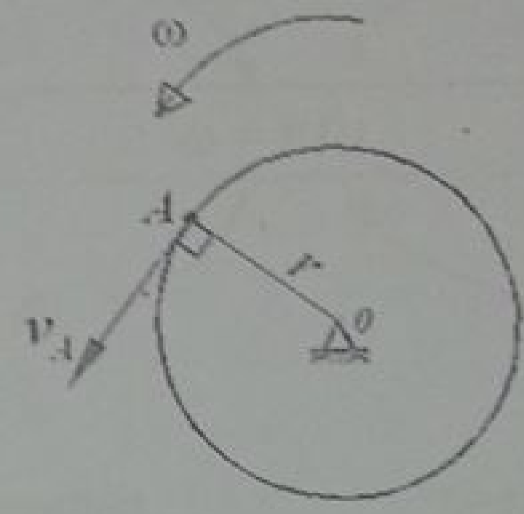
Cylinder rolls without slipping

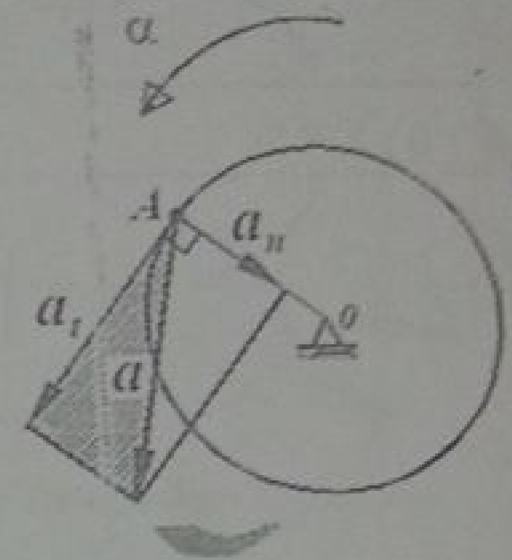


Ladder slips on a wall

(A) Rotation about fixed axis







Angular displacement:

$\theta = f(t)$ *rad*

$\theta = \theta^0 * \left(\frac{\pi}{180}\right) =$ *rad*

$\theta = (n)\, rev. * (2\pi) =$ *rad*

Angular velocity:

$\omega = \frac{d\theta}{dt} = \dot{\theta} =$ *rad/sec*

$\omega = (n)rpm * \left(\frac{2\pi}{60}\right) =$ *rad/sec*

Angular acceleration:

$\alpha = \frac{d\omega}{dt} = \dot{\omega} = \ddot{\theta} =$ *rad/sec²*

Displacement of a point:

$S_A = \theta * r =$ *m*

Velocity of a point:

$v_A = \omega * r =$ *m/sec*

Acceleration of a point:

$a_t = \alpha * r =$ m/sec^2

$a_n = \frac{v^2}{r} = \omega^2 . r =$ *m/sec*

$a = \sqrt{{a_t}^2 + {a_n}^2} =$ *m/sec*

$$\omega = \frac{d\theta}{dt} \qquad\qquad \alpha = \frac{d\omega}{dt}$$

$$\theta = f(t) \qquad \omega = f(t) \qquad \alpha = f(t)$$

$$\theta = \int \omega \, . dt \qquad\qquad \omega = \int \alpha \, . dt$$

$$c_2 = \theta_0 \qquad\qquad c_1 = \omega_0$$

Constant of Integration — *Constant of Integration*

The angular acceleration (α) may be given in these forms

1 − $\alpha = f(t)$ use the relation $\int \alpha . dt = \int d\omega$

2 − $\alpha = f(\theta)$ use the relation $\int \alpha . d\theta = \int \omega . d\omega$

3 − $\alpha = f(\omega)$ use the relation $\int d\theta = \int \omega . \frac{d\omega}{\alpha}$

4 − $\alpha = Constant$ use the relation $\alpha = \alpha_c$

If $\alpha = \alpha_c$ *so the angular velocity (ω) & angular displacement (θ) are;*

$$\omega = \int \alpha_c . dt = \alpha_c t + c_1 = \omega_0 + \alpha_c t \qquad \text{where } c_1 = \omega_0$$

$$\theta = \int \omega . dt = \omega_0 t + \frac{1}{2}\alpha_c t^2 + c_2 = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2 \qquad \text{where } c_2 = \theta_0$$

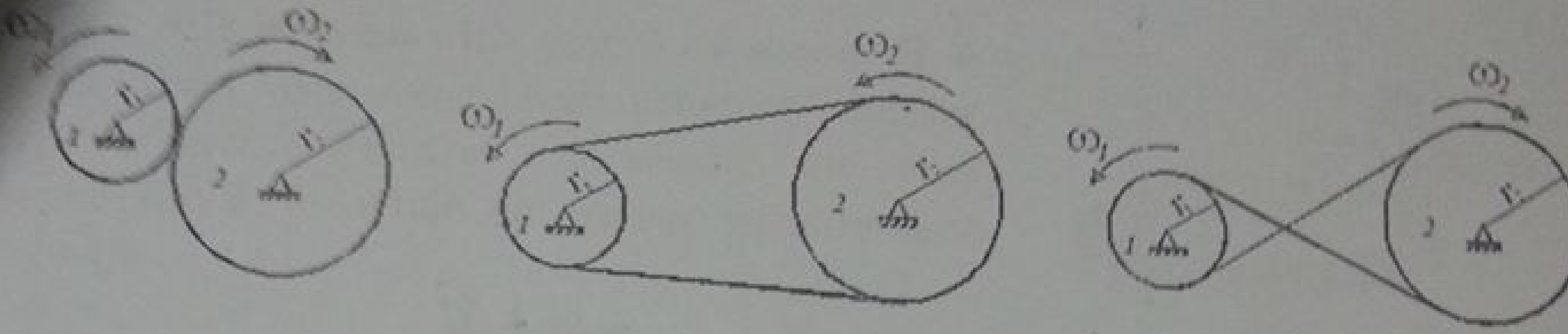
$$\omega^2 = \omega_0^{\,2} + 2\alpha_c(\theta - \theta_0)$$

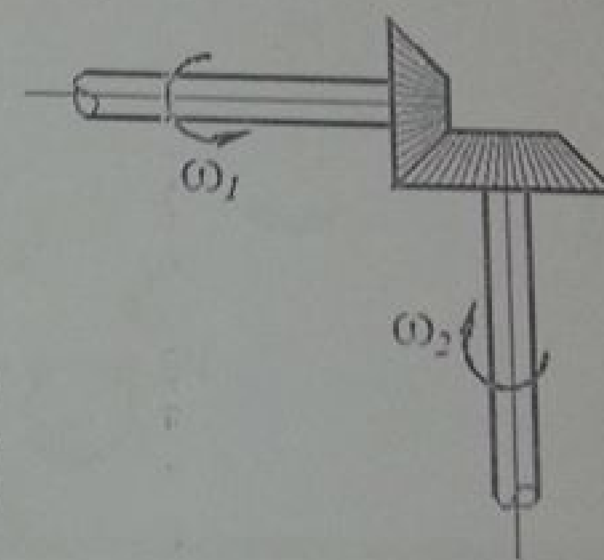
If $\omega = \omega_c$ *so the angular acceleration (α) & angular displacement (θ) are;*

$$\alpha = 0$$

$$\theta = \int \omega . dt = \omega_c t + c = \theta_0 + \omega_c t \qquad \text{where } c = \theta_0$$

Rotate by contact





$$\theta_1 * r_1 = \theta_2 * r_2$$

$$\omega_1 * r_1 = \omega_2 * r_2$$

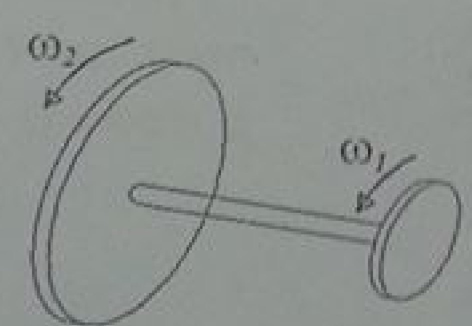
$$\alpha_1 * r_1 = \alpha_2 * r_2$$

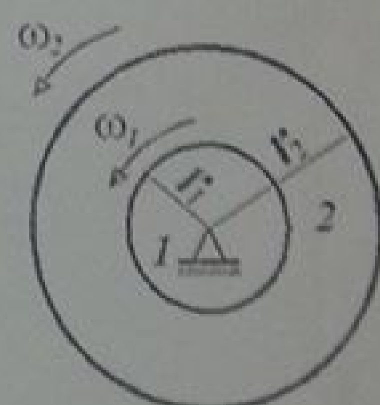
Two Pulleys Rotate on the same axis

$$\theta_1 = \theta_2$$

$$\omega_1 = \omega_2$$

$$\alpha_1 = \alpha_2$$



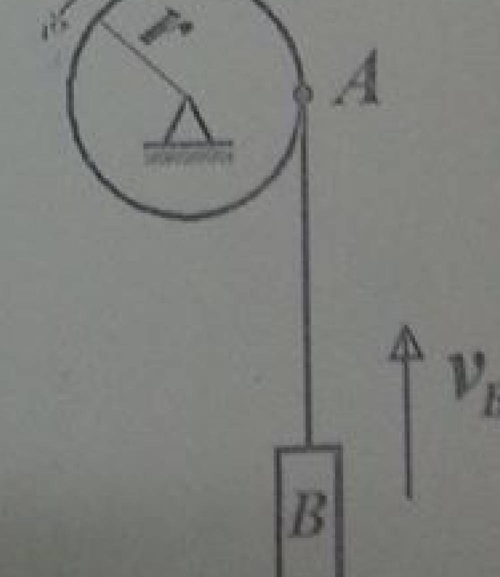


Rotation of Pulley with Translation of Block

$$S_B = S_A = \theta . r$$

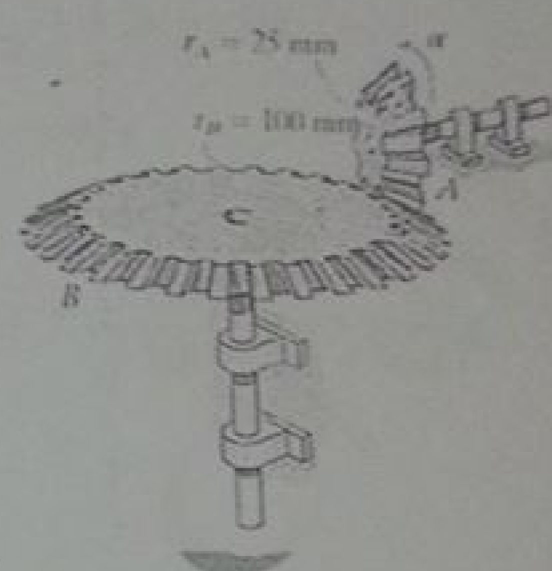
$$v_B = v_A = \omega . r$$

$$a_B = a_{t_A} = \alpha . r$$



Rotation Problems

1- Gear A is in mesh with gear B as shown. If A starts from rest and has constant angular acceleration $\alpha_A = 2\ rad/s^2$, determine the time needed for B to attain an angular velocity $\omega_B = 50\ rad/s$.



Solution

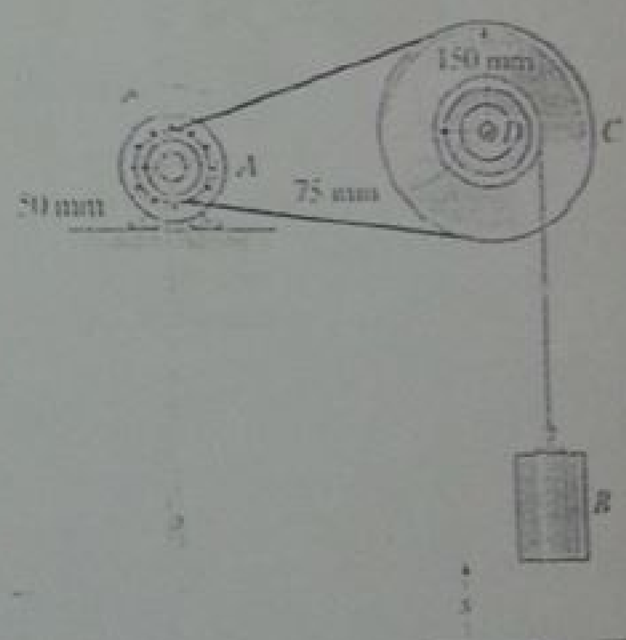
Two gears (A & B) rotate by contact

$\alpha_A * r_A = \alpha_B * r_B \quad \Rightarrow \quad \therefore \alpha_B = 2 * \left(\frac{25}{100}\right) = 0.5\ rad/s^2$

for const. acc. $\quad \omega = \omega_0 + \alpha_c t \quad \Rightarrow \quad 50 = 0 + 0.5\, t$

$\therefore t = 50/0.5 = 100\ sec$

2-Starting from rest when $S = 0$. Pulley A is given a constant angular acceleration $\alpha_A = 6\ rad/s^2$. Determine the speed of block B when it has risen to $S = 6\ m$. The pulley has an inner hub D which is fixed to C and turns with it.



Solution

Givens: $\quad \alpha_A = 6\ rad/s^2 \quad , \quad \theta_0 = \omega_0 = 0$

$r_A = 50\ mm \quad , \quad r_C = 150\ mm \quad \& \quad r_D = 75\ mm$

Req.: $\quad v_B = ?? \quad \Rightarrow \quad at\ \ S_B = 6\ m$

for const. acc. $\quad \omega^2 = {\omega_0}^2 + 2\alpha_c(\theta - \theta_0)$

$\omega_A^2 = 0 + 2 * 6 * (\theta_A - 0) \quad \Rightarrow \quad \omega_A = \sqrt{12\,\theta_A}$ ....(1)

Rotation of Pulley (D) with Translation of Block (B)

$S_B = 6\ m = \theta_D * r_D \quad \Rightarrow \quad \therefore \theta_D = \theta_C = \frac{6}{75 * 10^{-3}} = 80\ rad$

Two pulleys (A & C) rotate by contact

$$\theta_A * r_A = \theta_C * r_C \quad \Rightarrow \quad \therefore \theta_A = 80 * \left(\frac{150}{50}\right) = 240\ rad$$

Sub. into Eq. (1) yields
$$\therefore \omega_A = \sqrt{12 * 240} = 53.7\ rad/s$$

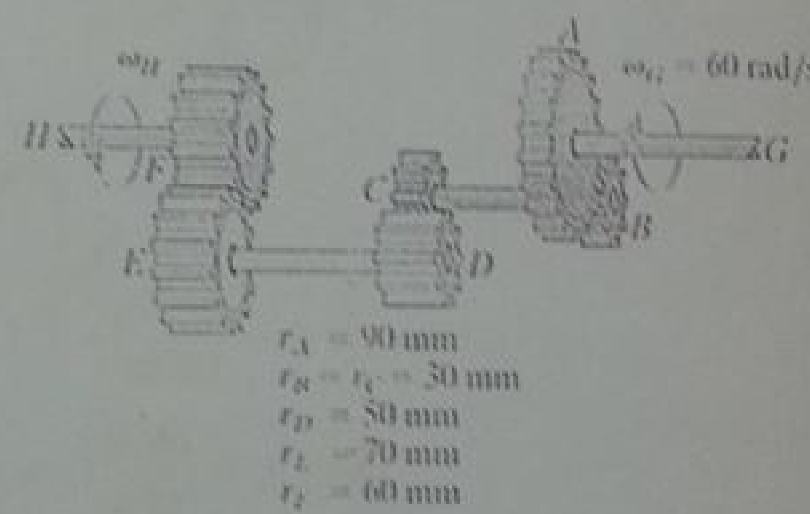
Two pulleys (A & C) rotate by contact

$$\omega_A * r_A = \omega_C * r_C \quad \Rightarrow \quad \therefore \omega_C = \omega_D = 53.7 * \left(\frac{50}{150}\right) = 17.9\ rad/s$$

The velocity of block B when it raised $S_B = 6\ m$

$$\therefore v_B = \omega_D * r_D = 17.9 * (75 * 10^{-3}) = 1.34\ m/s$$

3- The operation of "reverse" for a three-speed automotive transmission is illustrated schematically in the figure. If the crank G is turning with an angular speed of 60 rad/s, determine the angular speed of the drive shaft H. Each of gears rotates about a fixed axis. Note that gear A and B, C and D, E and F are in mesh. The radii of each of these gears are reported in the figure.



Solution

$$\omega_G = \omega_A = 60\ rad/s$$

Two gears (A & B) rotate by contact

$$\omega_A * r_A = \omega_B * r_B \quad \Rightarrow \quad \therefore \omega_B = \omega_C = 60 * \left(\frac{90}{30}\right) = 180\ rad/s$$

Two gears (C & D) rotate by contact

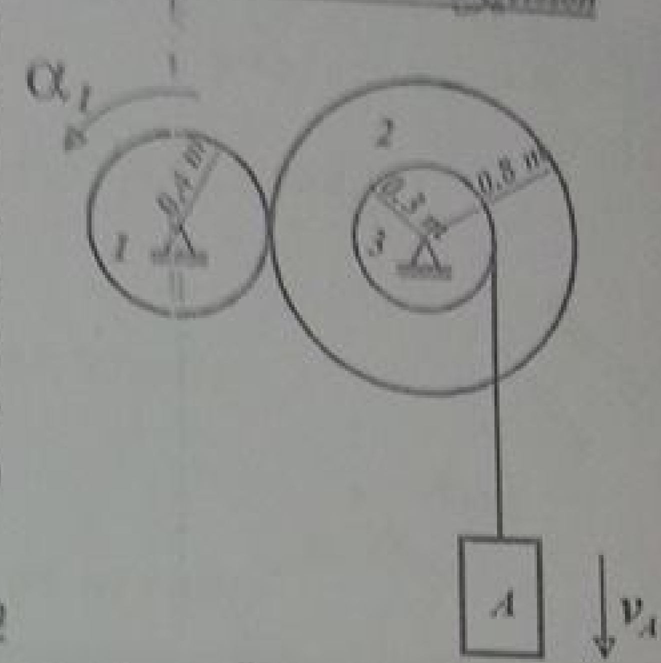
$$\omega_C * r_C = \omega_D * r_D \quad \Rightarrow \quad \therefore \omega_D = \omega_E = 180 * \left(\frac{30}{50}\right) = 108\ rad/s$$

Two gears (E& F) rotate by contact

$$\omega_E * r_E = \omega_F * r_F \quad \Rightarrow \quad \therefore \omega_F = \omega_H = 108 * \left(\frac{70}{60}\right) = 126\ rad/s$$

$$\therefore \omega_F = \omega_E = 46.22\ rad/s$$

4-In the gear train shown in figure, the gear (1) of radius of $R_1 = 0.4\ m$, starts to rotate from rest with constant angular acceleration $\alpha_1 = 1.5\ r/s^2$ counterclockwise. The gear (2) has outer radius of $R_2 = 0.4\ m$ and inner radius of $r_3 = 0.3\ m$. Block A is held at the end of the rope. After 4 sec from the start of motion determine: (a) the angular velocity and angular acceleration of gear (3). (b) the velocity and acceleration of block A.



Solution

Givens: $\alpha_1 = 1.5\ r/s^2$, $R_1 = 0.4\ m$, $R_2 = 0.8\ m$ & $r_3 = 0.3\ m$

Req.: $\omega_3 = ??$, $\alpha_3 = ??$, $v_A = ??$ & $a_A = ?? \rightarrow at\ t = 4\ sec$

For constant acceleration:

$$\omega_1 = (\omega_0)_1 + \alpha_1 t = 0 + 1.5 * 4 = 6\ rad/s$$

Two gears (1 & 2) rotate by contact

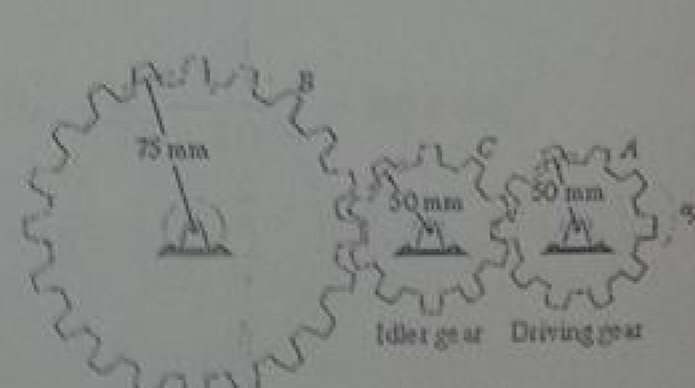
$$\alpha_1 * R_1 = \alpha_2 * R_2 \quad \Longrightarrow \quad \therefore \alpha_2 = \alpha_3 = 1.5 * \left(\frac{0.4}{0.8}\right) = 0.75\ rad/s^2$$

$$\omega_1 * R_1 = \omega_2 * R_2 \quad \Longrightarrow \quad \therefore \omega_2 = \omega_3 = 6 * \left(\frac{0.4}{0.8}\right) = 3\ rad/s$$

The velocity and acceleration of block A when t = 4 sec are

$$\therefore v_A = \omega_3 * r_3 = 3 * 0.3 = 0.9\ m/s$$

5-When only two gears are in mesh, the driving gear A and the driven gear B will always turn in opposite directions. In order to get them to turn in the same direction an idler gear C is used. In the case shown, determine the angular velocity of gear B when $t = 5\ sec$, if gear A starts from rest and has an angular acceleration of $\alpha_A = (3t + 2)\ rad/s^2$, where t is in seconds.



Solution

$$\omega_A = \int (3t+2)\,dt = 1.5\,t^2 + 2\,t + \omega_{0_A} = 1.5\,t^2 + 2\,t \qquad \text{where, } \omega_{0_A} = 0$$

$$at \quad t = 5\,sec \quad \Longrightarrow \quad \therefore \omega_A = 1.5 * (5)^2 + 2 * (5) = 47.5\,rad/s$$

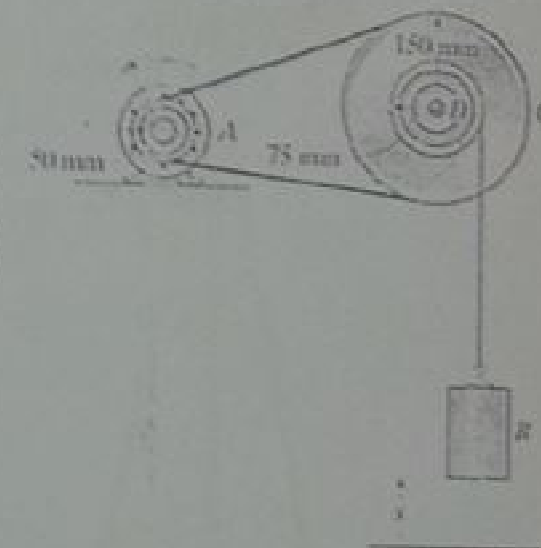
<u>Two gears (A & C) rotate by contact</u>

$$\omega_A * r_A = \omega_C * r_C \quad \Longrightarrow \quad \therefore \omega_C = 47.5 * \left(\frac{50}{50}\right) = 47.5\,rad/s$$

<u>Two gears (C& B) rotate by contact</u>

$$\omega_C * r_C = \omega_B * r_B \quad \Longrightarrow \quad \therefore \omega_B = 47.5 * \left(\frac{50}{75}\right) = 31.67\,rad/s$$

6- Starting from rest when $S = 0$. Pulley A is given a constant angular acceleration $\alpha_A = (6\theta)\,rad/s^2$, where θ is in radians. Determine the speed of block B when it has risen to $S = 6\,m$.



<u>Solution</u>

<u>Givens:</u> $\alpha_A = (6\theta)\,rad/s^2$, $\theta_0 = \omega_0 = 0$

$r_A = 50\,mm$, $r_C = 150\,mm$ & $r_D = 75\,mm$

<u>Req.:</u> $v_B = ??$ $\Longrightarrow$ at $S_B = 6\,m$

$$\int_{\theta_0=0}^{\theta} (6\,\theta)\,d\theta = \int_{\omega_0=0}^{\omega} \omega.\,d\omega \Longrightarrow \frac{6}{2}\,\theta_A{}^2 = \frac{\omega_A{}^2}{2} \quad \Longrightarrow \quad \omega_A = \sqrt{6\,\theta_A{}^2} \quad .....(1)$$

<u>Rotation of Pulley (D) with Translation of Block (B)</u>

$$S_B = 6\,m = \theta_D * r_D \quad \Longrightarrow \quad \therefore \theta_D = \theta_C = \frac{6}{75 * 10^{-3}} = 80\,rad$$

<u>Two pulleys (D & C) rotate on the same axis</u>

$$\therefore \theta_C = \theta_D = 80\,rad$$

<u>Two pulleys (A & C) rotate by contact</u>

$$\theta_A * r_A = \theta_C * r_C \quad \Longrightarrow \quad \therefore \theta_A = 80 * \left(\frac{150}{50}\right) = 240\,rad$$

<u>Sub. into Eq. (1) yields</u> $\therefore \omega_A = \sqrt{6 * 240^2} = 587.8\,rad/s$

Two pulleys (A & C) rotate by contact

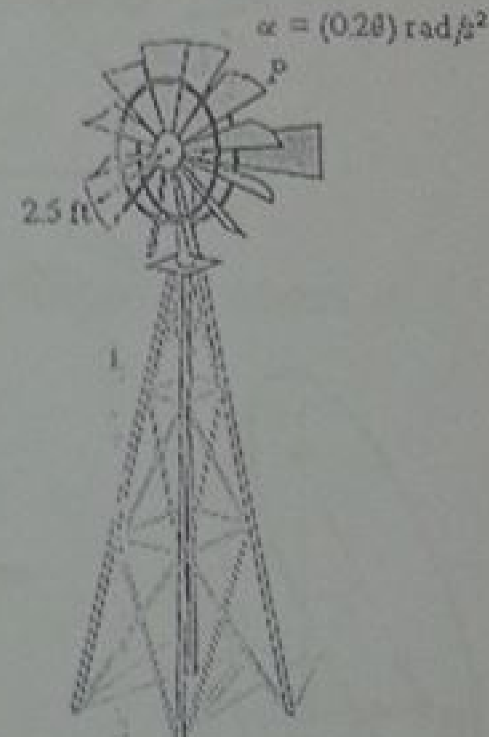
$\omega_A * r_A = \omega_C * r_C \qquad \Longrightarrow \qquad \therefore \omega_C = 587.8 * \left(\frac{50}{150}\right) = 195.96\, rad/s$

Two pulleys (D & C) rotate on the same axis

The velocity of block B when it raised $S_B = 6\ m$

$\therefore v_B = \omega_D * r_D = 195.96 * (75 * 10^{-3}) = 14.7\ m/s$

7-During a gust of wind, the blades of the windmill are given an angular acceleration of $\alpha = (0.2\,\theta)\ rad/s^2$, where θ is in radians. If initially the blades have an angular velocity of $\omega = 5\ rad/s$, determine the speed of points P, located at the tip of one of the blades, just after the blade has turned $2\ rev$.



Solution

Givens:

$\alpha = (0.2\,\theta)\ rad/s^2 \qquad \& \qquad \omega_0 = 5\ rad/s$

$r_P = 2.5\ ft$

Req.: $\qquad v_P = ?? \quad \Longrightarrow \quad after \quad \theta_P = 2\, rev = 2 * 2\pi = 12.56\, rad$

$\alpha = f(\theta) \qquad$ *use the relation* $\qquad \int \alpha . d\theta = \int \omega . d\omega$

$$\int_{\theta_0=0}^{\theta} (0.2\,\theta)\, d\theta = \int_{\omega_0=5}^{\omega} \omega . d\omega \qquad \frac{0.2\,\theta^2}{2} = \frac{\omega^2 - 5^2}{2}$$

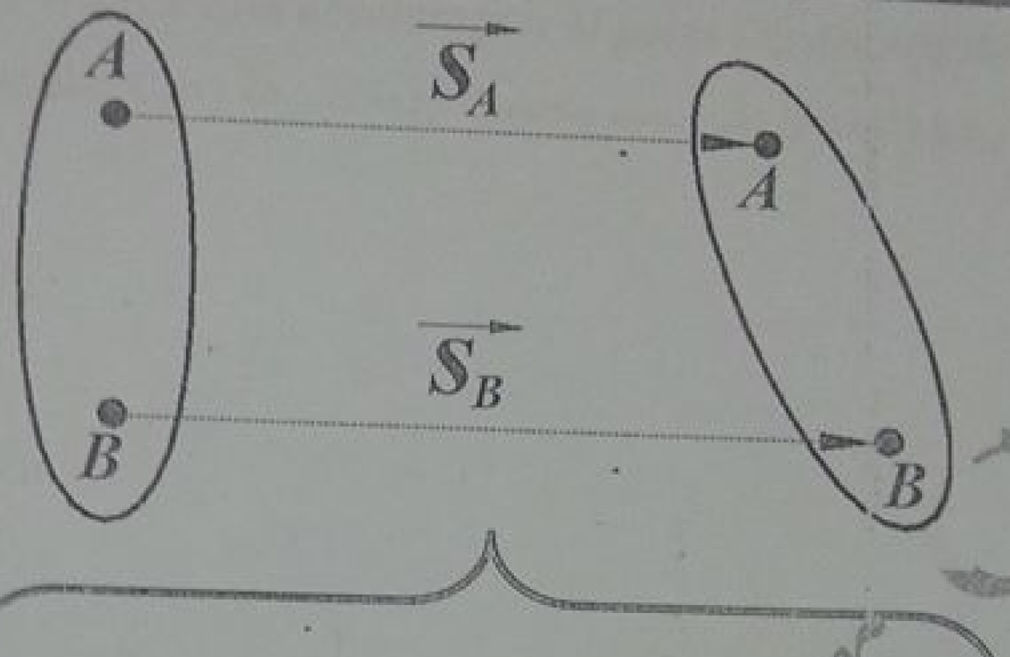
$\omega^2 = 0.2\,\theta^2 + 25$

$\therefore \omega = \sqrt{0.2 * (12.56)^2 + 25} = 7.52\ rad/s$

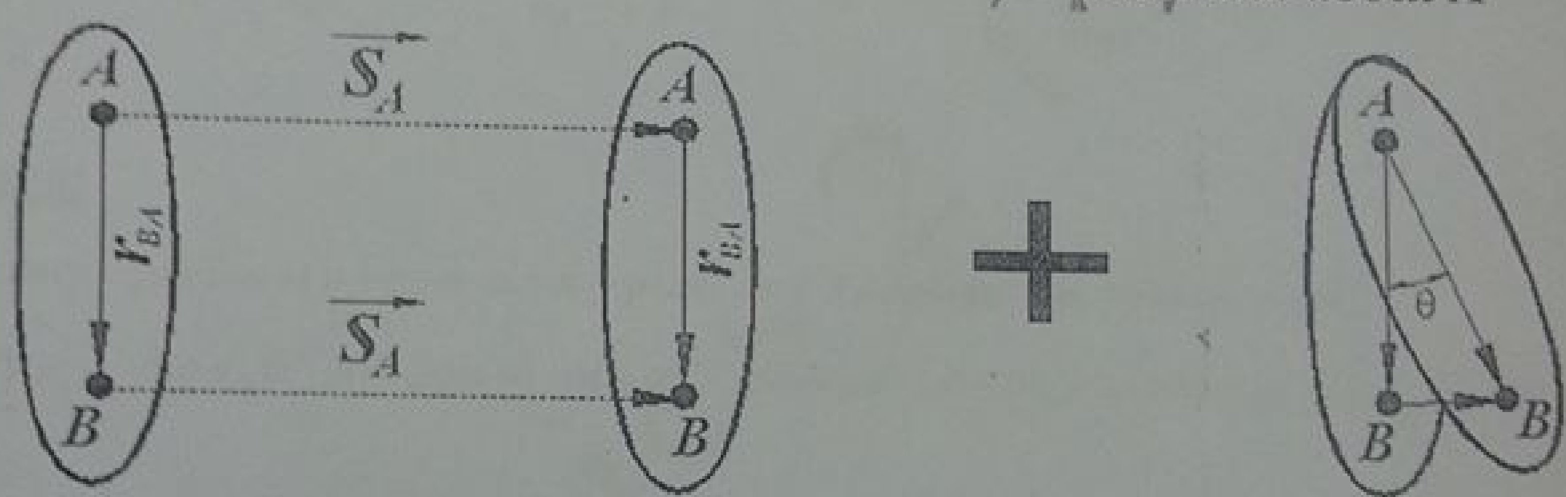
The velocity of point P after $\theta_P = 2\ rev$

$\therefore v_P = \omega * r_P = 7.52 * 2.5 = 18.8\ ft/s$

(B) Relative Motion Analysis (G.P.M.)



Translation + Rotation about A



If displacement, velocity and acceleration of point (A) are given.

And displacement, velocity and acceleration of point (B) are required.

تحديد ازاحة وسرعة وعجله نقطه A على الجسم بمعلومية ازاحة وسرعة وعجله نقطه اخرى B على الجسم.

Displacement:

$$\overrightarrow{S_B} = \overrightarrow{S_A} + \vec{\theta}\ X\ \overrightarrow{r_{BA}} \qquad \dots\dots\dots\dots\dots\dots\dots (1)$$

Velocity:

$$\overrightarrow{v_B} = \overrightarrow{v_A} + \vec{\omega}\ X\ \overrightarrow{r_{BA}} \qquad \dots\dots\dots\dots\dots\dots\dots (2)$$

Acceleration:

$$\overrightarrow{a_B} = \overrightarrow{a_A} + \left[\underbrace{\vec{\alpha}\ X\ \overrightarrow{r_{BA}}}_{a_t} - \underbrace{\omega^2 * \overrightarrow{r_{BA}}}_{a_n}\right] \qquad \dots\dots\dots\dots\dots\dots\dots (3)$$

If displacement, velocity and acceleration of point (B) are given.

And displacement, velocity and acceleration of point (A) are required.

تحديد ازاحة وسرعة وعجله نقطه B على الجسم بمعلومية ازاحة وسرعة و عجله نقطه اخرى A على الجسم.

Displacement:

$$\overrightarrow{S_A} = \overrightarrow{S_B} + \vec{\theta}\ X\ \overrightarrow{r_{AB}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Velocity:

$$\overrightarrow{v_A} = \overrightarrow{v_B} + \vec{\omega}\ X\ \overrightarrow{r_{AB}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Acceleration:

$$\overrightarrow{a_A} = \overrightarrow{a_B} + \left[\underbrace{\vec{\alpha}\ X\ \overrightarrow{r_{AB}}}_{a_t} - \underbrace{\omega^2 * \overrightarrow{r_{AB}}}_{a_n}\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Notes:

1- Every equation is a vector in x-y plane and it contains two unknowns only.

جميع المعادلات السابقه هى معادلات اتجاهيه فى المستوى $x - y$ وعدد المجاهيل داخل كل معادله لا يزيد عن مجهولين.

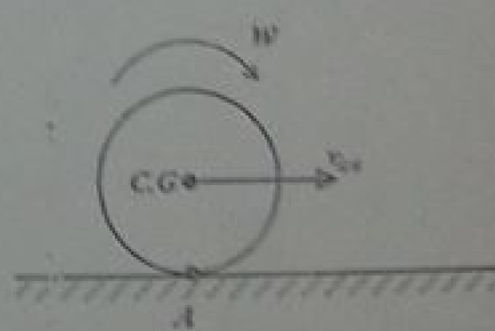
2- $\overrightarrow{\omega_{AB}} = \overrightarrow{\omega_{BA}} = \omega_{AB}\ \vec{k}$ — C.C.W ↺ الدوران ضد عقارب الساعه موجب

3- $\overrightarrow{\alpha_{AB}} = \overrightarrow{\alpha_{BA}} = \alpha_{AB}\ \vec{k}$ — C.C.W ↺ الدوران ضد عقارب الساعه موجب

4- If no slipping on the surface then;

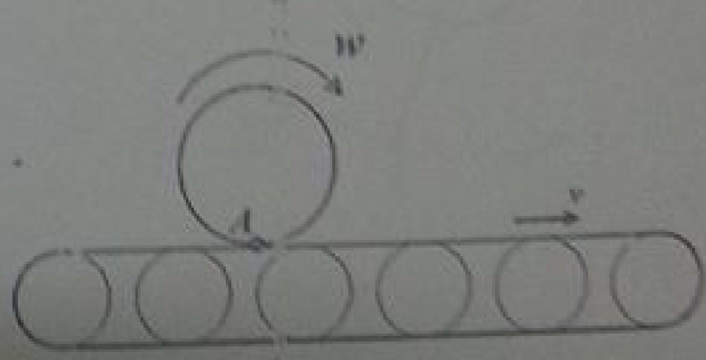
$\overrightarrow{v_A} = 0$ & $\overrightarrow{a_A} = (\omega^2 . r)\ \vec{j}$

$\overrightarrow{v_{C.G}} = (\omega . r)\ \vec{i}$ & $\overrightarrow{a_{C.G}} = (\alpha . r)\ \vec{i}$



5- If no slipping on the moving belt then;

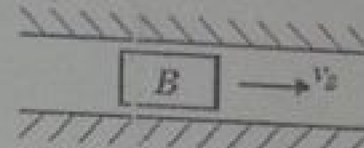
$\overrightarrow{v_A} = v_{belt}\ \vec{i}$



Translation as a vector:

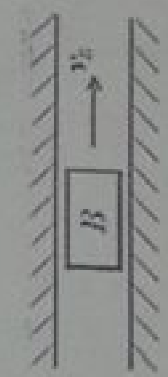
(1) $\overrightarrow{v_B} = v_B \, \vec{i}$

$\overrightarrow{a_B} = a_B \; \vec{i}$



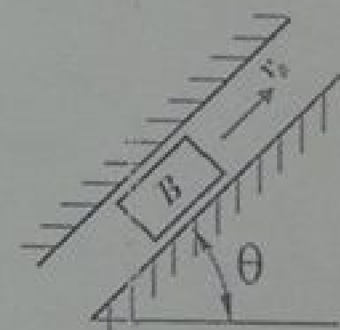
(2) $\overrightarrow{v_B} = v_B \; \vec{j}$

$\overrightarrow{a_B} = a_B \; \vec{j}$



(3) $\overrightarrow{v_B} = (v_B \cos\theta)\,\vec{i} + (v_B \sin\theta)\,\vec{j}$

$\overrightarrow{a_B} = (a_B \cos\theta)\,\vec{i} + (a_B \sin\theta)\,\vec{j}$

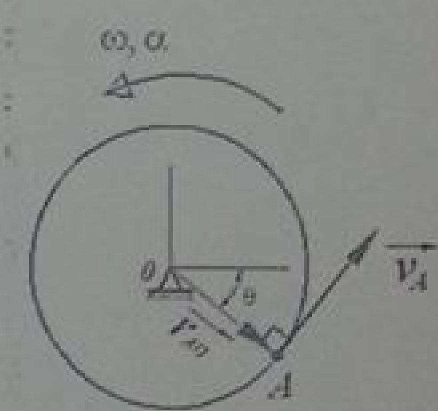


Rotation as a vector:

Center point of Rotation:-

$\overrightarrow{v_0} = 0$

$\overrightarrow{a_0} = 0$



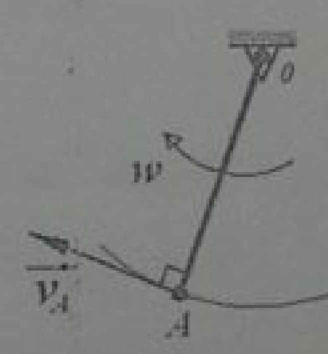
Point at outer rim of disk:-

$\overrightarrow{v_A} = \vec{\omega} \times \overrightarrow{r_{AO}}$

$\overrightarrow{a_A} = \vec{\alpha} \times \overrightarrow{r_{AO}} - \omega^2 * \overrightarrow{r_{AO}}$

Where,

$\overrightarrow{r_{AO}} = \pm(r\cos\theta)\,\vec{i} \pm (r\sin\theta)\,\vec{j}$



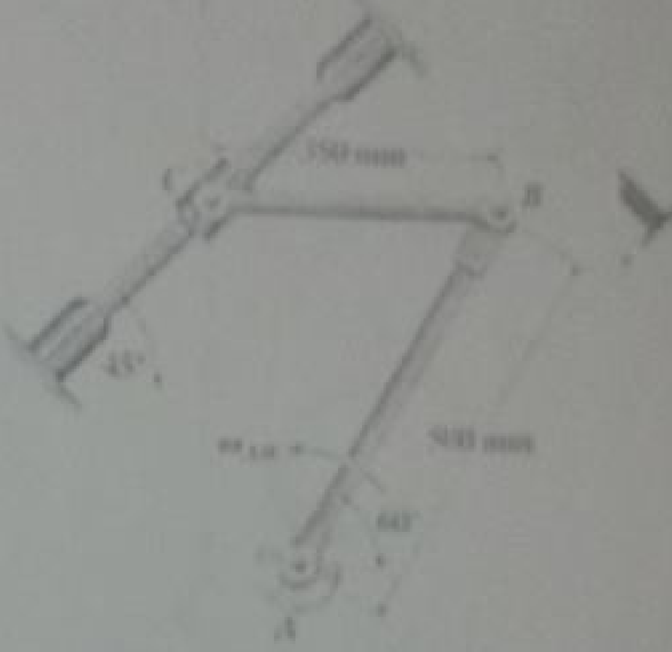
$\vec{\omega} = \pm\omega\,\vec{k}$ & $\vec{\alpha} = \pm\alpha\,\vec{k}$

c.c.w ↶ = +ve & c.w ↷ = −ve

And, $\vec{k} \times \vec{i} = \vec{j}$ & $\vec{k} \times \vec{j} = -\vec{i}$

Velocity Problems

1- If the collar C is moving downward to the left at $v_C = 8\ m/s$, determine the angular velocity of link AB at the instant



Solution

Givens:

$l_{AB} = 0.5\ m$, $l_{BC} = 0.35\ m$,

$v_c = 8\ m/s$ ✓

Req : $\omega_{AB} = ??$

Velocity analysis:

$\overrightarrow{v_C} = \overrightarrow{v_B} + \overrightarrow{v_{CB}} = \overrightarrow{v_B} + \overrightarrow{\omega_{CB}} \times \overrightarrow{r_{CB}}$ ....(1)

where, $\overrightarrow{v_C} = -8\cos 45\ \hat{i} - 8\sin 45\ \hat{j}$

$= -4\sqrt{2}\ \hat{i} - 4\sqrt{2}\ \hat{j}$ ....(2)

and, $\overrightarrow{v_B} = \overrightarrow{\omega_{AB}} \times \overrightarrow{r_{BA}} = \omega_{AB}\ \vec{k} \times (0.5\cos 60\ \hat{i} + 0.5\sin 60\ \hat{j})$

$= 0.25\,\omega_{AB}\ \hat{j} - 0.25\sqrt{3}\,\omega_{AB}\ \hat{i}$ ....(3)

and, $\overrightarrow{v_{CB}} = \overrightarrow{\omega_{CB}} \times \overrightarrow{r_{CB}} = \omega_{CB}\ \vec{k} \times (-0.35\ \hat{i})$

$= -0.35\,\omega_{CB}\ \hat{j}$ ....(4)

Sub. Eq. (2), (3) & (4) into Eq. (1) yields;

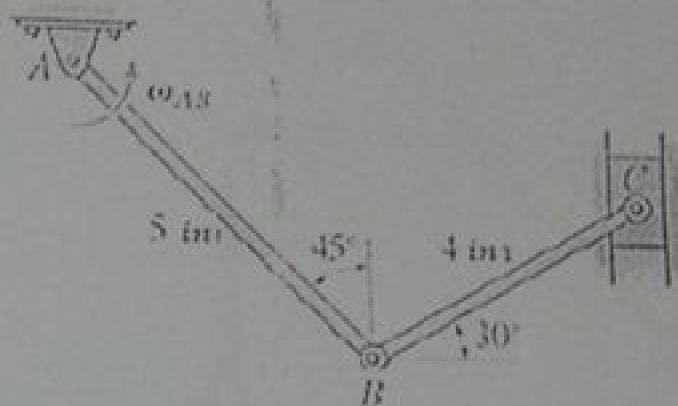
$(-4\sqrt{2}\ \hat{i} - 4\sqrt{2}\ \hat{j}) = (0.25\,\omega_{AB}\ \hat{j} - 0.25\sqrt{3}\,\omega_{AB}\ \hat{i}) + (-0.35\,\omega_{CB}\ \hat{j})$

Equating the left and right coefficients of the same direction i.e. "the same unit vector"

i - equation $-4\sqrt{2} = -0.25\sqrt{3}\,\omega_{AB} \Rightarrow \therefore \omega_{AB} = 13.06\ rad/s$ c.c.w

j - equation $-4\sqrt{2} = 0.25\,\omega_{AB} - 0.35\,\omega_{CB} \Rightarrow \therefore \omega_{CB} = 25.5\ rad/s$ c.c.w

2- Determine the velocity of block C at the instant shown if the link AB is rotating at $\omega_{AB} = 8\ rad/s$.



Solution

Givens:

$l_{AB} = 5\ in$, $l_{BC} = 4\ in$ & $\omega_{AB} = 8\ rad/s$ c.c.w

Req.: $v_C = ??$ of block C

Velocity analysis:

$$\overrightarrow{v_C} = \overrightarrow{v_B} + \overrightarrow{\omega_{CB}} X \overrightarrow{r_{CB}}$$

where, $\overrightarrow{v_B} = \overrightarrow{\omega_{BA}} X \overrightarrow{r_{BA}} = 8\ \vec{k}\ X\ (5 \sin 45\ \vec{i} - 5 \cos 45\ \vec{j})$

$$= 20\sqrt{2}\ \vec{j} + 20\sqrt{2}\ \vec{i}$$

$$v_C\,\vec{j} = (20\sqrt{2}\ \vec{j} + 20\sqrt{2}\ \vec{i}) + \omega_{CB}\ \vec{k}\ X\ (4 \cos 30\ \vec{i} + 4 \sin 30\ \vec{j})$$

$$v_C\,\vec{j} = (20\sqrt{2}\ \vec{j} + 20\sqrt{2}\ \vec{i}) + (2\sqrt{3}\ \omega_{CB}\ \vec{j} - 2\ \omega_{CB}\ \vec{i})$$

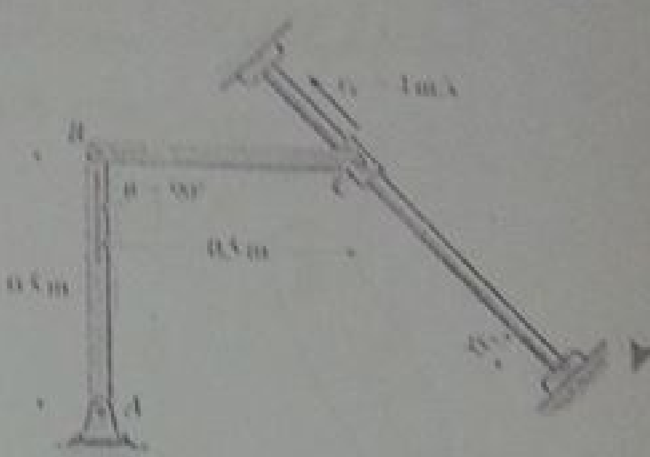
i - equation $\quad 0 = 20\sqrt{2} - 2\ \omega_{CB}$

$\therefore\ \omega_{CB} = 14.14\ rad/s$ ↶ c.c.w

j - equation $\quad v_C = 20\sqrt{2} + 2\sqrt{3}\ \omega_{CB}$

$\therefore\ v_C = 77.3\ in/s$ ↑

3- Determine the angular velocity of the link AB at the instant shown if the collar C has velocity $v_c = 4\ m/s$ as shown.



Solution

Givens:

$l_{AB} = 0.5\ m$, $l_{BC} = 0.5\ m$,

$v_c = 4\ m/s$ ↖

Req.: $\omega_{AB} = ??$

Velocity analysis:

$$\overrightarrow{v_C} = \overrightarrow{v_B} + \overrightarrow{v_{CB}} = \overrightarrow{v_B} + \overrightarrow{\omega_{CB}} \times \overrightarrow{r_{CB}} \quad \text{.......................(1)}$$

where, $$\overrightarrow{v_C} = -4\cos 45\ \vec{i} + 4\sin 45\ \vec{j}$$

$$= -2\sqrt{2}\ \vec{i} + 2\sqrt{2}\ \vec{j} \quad \text{.......................(2)}$$

and, $$\overrightarrow{v_B} = \overrightarrow{\omega_{AB}} \times \overrightarrow{r_{BA}} = \omega_{AB}\ \vec{k} \times (0.5\ \vec{j})$$

$$= -0.5\ \omega_{AB}\ \vec{i} \quad \text{.......................(3)}$$

and, $$\overrightarrow{v_{CB}} = \overrightarrow{\omega_{CB}} \times \overrightarrow{r_{CB}} = \omega_{CB}\ \vec{k} \times (0.5\ \vec{i})$$

$$= 0.5\ \omega_{CB}\ \vec{j} \quad \text{.......................(4)}$$

Sub. Eq. (2), (3) & (4) into Eq. (1) yields;

$$(-2\sqrt{2}\ \vec{i} + 2\sqrt{2}\ \vec{j}) = (-0.5\ \omega_{AB}\ \vec{i}) + (0.5\ \omega_{CB}\ \vec{j})$$

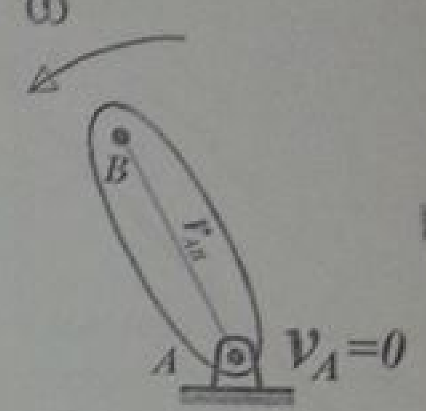
Equating the left and right coefficients of the same direction i.e. "the same unit vector"

i - equation $\quad -2\sqrt{2} = -0.5\ \omega_{AB} \quad \Longrightarrow \quad \therefore \omega_{AB} = 4\sqrt{2}\ rad/s$ ↶ c.c.w

j - equation $\quad 2\sqrt{2} = 0.5\ \omega_{CB} \quad \Longrightarrow \quad \therefore \omega_{CB} = 4\sqrt{2}\ rad/s$ ↶ c.c.w

Relative Velocity Using Instantaneous Center (I.C)

Instantaneous Center (I.C) is a position at which the velocity is zero.



The velocity equation in G.P.M is

$$\overrightarrow{v_B} = \overrightarrow{v_A} + \overrightarrow{\omega_{BA}} \times \overrightarrow{r_{BA}}$$

$$\overrightarrow{v_A} = 0$$

$$\therefore v_B = \omega_{BA} \cdot r_{BA}$$

تحويل الحركه العامه فى المستوى "أنتقاليه+دورانيه" الى حركه دورانيه فقط عن طريق مركز الدوران اللحظى

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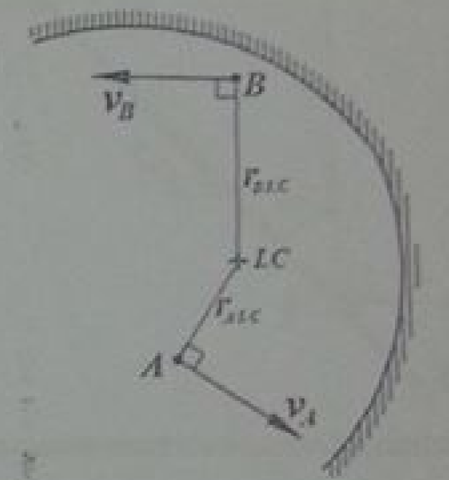
*The directions of two non-parallel velocities are known.



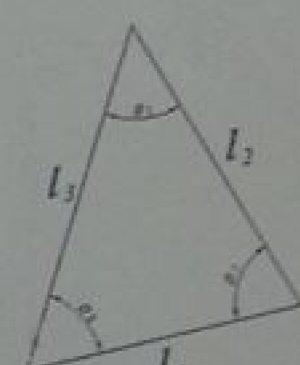
givens: dir. of v_A & v_B

$r_{A I.C} \perp v_A$ & $r_{B I.C} \perp v_B$

Notes: ملاحظـــــات هامـــــه

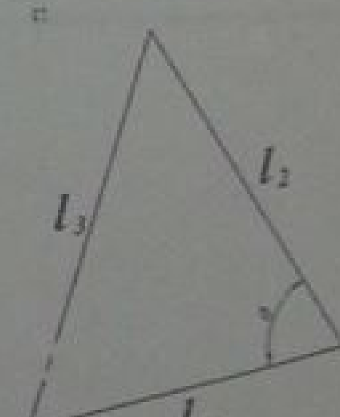
1-Sine Rule

$$\frac{l_1}{\sin\theta_1} = \frac{l_2}{\sin\theta_2} = \frac{l_3}{\sin\theta_3}$$



2-Cosine Rule

$$l_3^{\,2} = l_1^{\,2} + l_2^{\,2} - 2l_1 l_2 \cos\theta$$

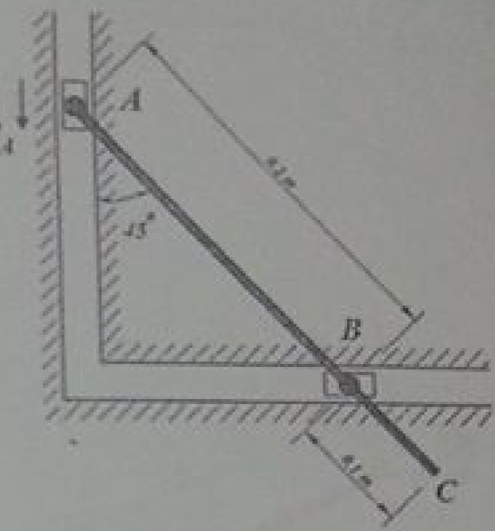


Special case if $\theta = 90^o$

$$\therefore l_3^{\,2} = l_1^{\,2} + l_2^{\,2}$$

Velocity Problems using Instantaneous Center (I.C)

1- The links shown in the figure is guided by two blocks at A and B, which move in the fixed slots. If the velocity of A is $v_A = 3\ m/s$ downward, determine the angular velocity of link AB and the linear velocity of block B at the instant $\theta = 45°$. Use instantaneous center (I.C).



Solution

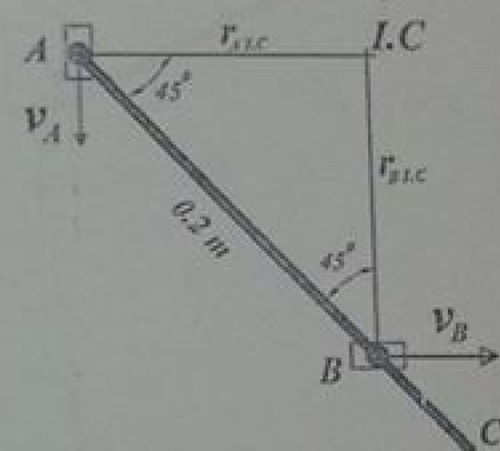
$$\therefore r_{A.IC} = r_{B.IC} = 0.1\sqrt{2}\ m$$

$$v_A = \omega_{AB} \cdot r_{A.IC} = 3\ m/s$$

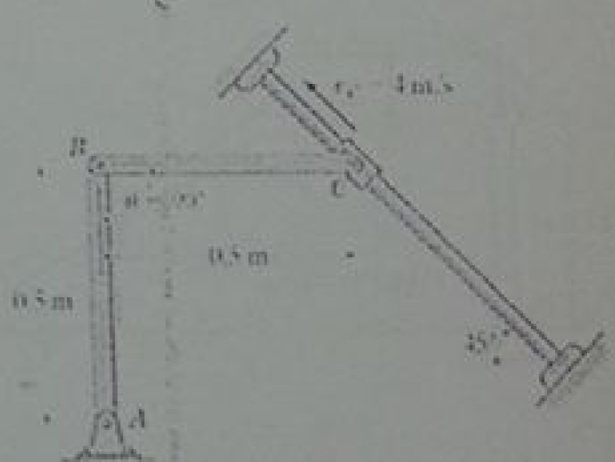
$$\therefore \omega_{BC} = \frac{3}{0.1\sqrt{2}} = 21.21\ rad/s \ \circlearrowleft$$

$$\therefore v_B = \omega_{AB} \cdot r_{B.IC} = 21.21 * 0.1\sqrt{2} = 3\ m/s \ \rightarrow$$

$$And \ \therefore v_C = \omega_{AB} \cdot r_{C.IC} = 21.21 * \sqrt{0.05} = 4.74\ m/s \ \nearrow$$



2- Determine the angular velocity of the link AB at the instant shown if the collar C has velocity $v_c = 4\ m/s$ as shown. Use instantaneous center (I.C).



Solution

$$\frac{r_{B.IC}}{\sin 45} = \frac{r_{C.IC}}{\sin 90} = \frac{0.5}{\sin 45}$$

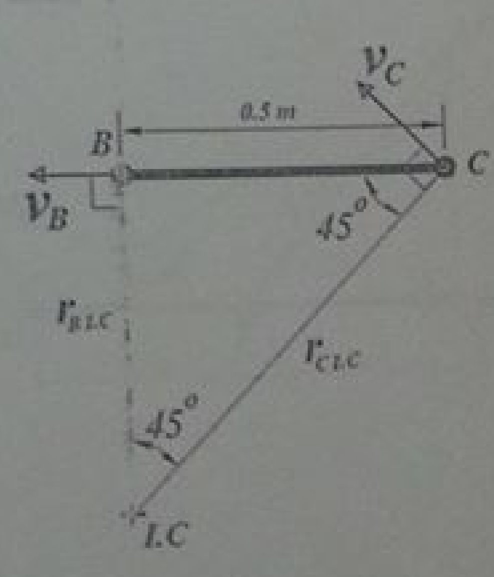
$$\therefore r_{B.IC} = 0.5\ m \quad \& \quad r_{C.IC} = 0.5\sqrt{2}\ m$$

$$v_C = \omega_{BC} \cdot r_{C.IC} = 4\ m/s$$

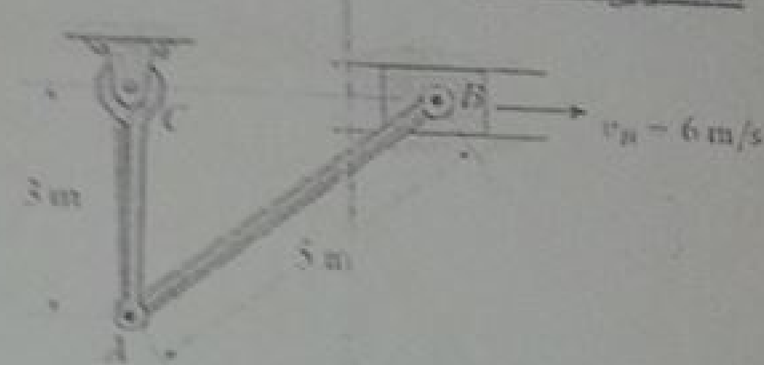
$$\therefore \omega_{BC} = \frac{4}{0.5\sqrt{2}} = 4\sqrt{2}\ rad/s \ \circlearrowleft$$

$$v_B = \omega_{BC} \cdot r_{B.IC} = \omega_{AB} \cdot r_{AB}$$

$$\therefore \omega_{AB} = 4\sqrt{2} * (0.5) = 4\sqrt{2}\ rad/s \ \circlearrowleft$$



3- The slider block B is moving to the right with velocity is $v_B = 6\ m\ rad/s$, at the instant shown. Determine the angular velocity of link AB & AC and the velocity of point A at this instant.



Solution

From graph $v_C \parallel v_B$

$\therefore r_{B.IC} = r_{A.IC} = \infty$

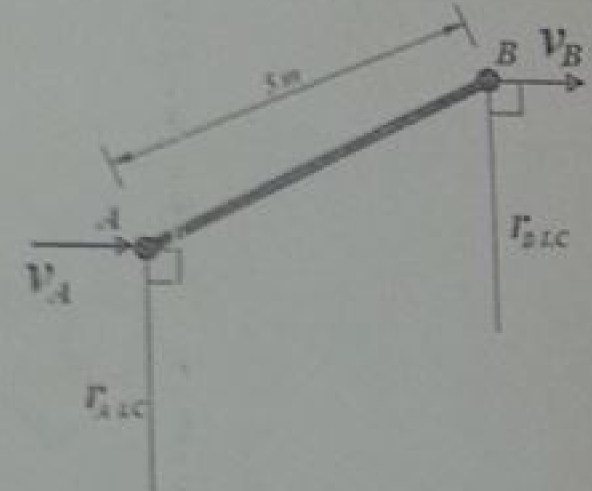
$v_B = \omega_{AB} \cdot r_{B.IC} = 6\ m/s$

$\therefore \omega_{AB} = \dfrac{6}{\infty} = 0$ i.e. no rotation at this instant

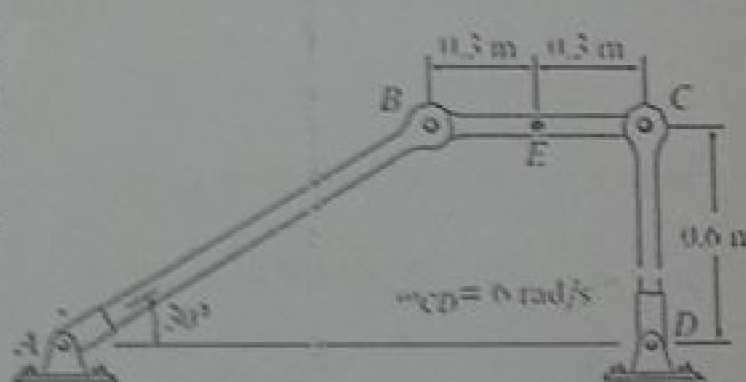
The link AB moves translation only at this instant

$\therefore v_A = v_B = 6\ m/s$

$v_A = \omega_{AC} \cdot r_{AC} \implies \therefore \omega_{AB} = \dfrac{6}{3} = 2\ rad/s$



4- If link CD has an angular velocity of $\omega_{CD} = 6\ rad/s$, determine the velocity of point E on link BC and the angular velocity of link AB at the instant shown. Use instantaneous center (I.C).



Solution

$$\frac{r_{B.IC}}{\sin 90} = \frac{r_{C.IC}}{\sin 30} = \frac{0.6}{\sin 60}$$

$\therefore r_{B.IC} = 0.7\ m$ & $r_{C.IC} = 0.35\ m$

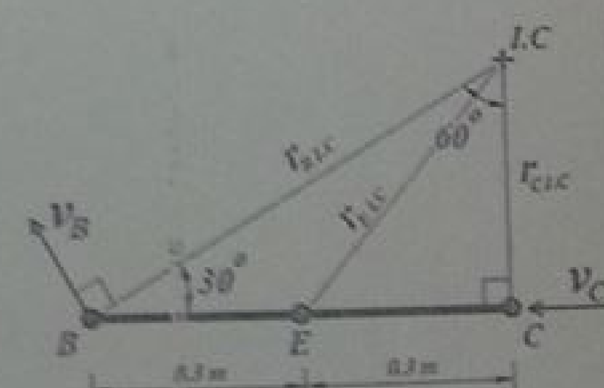
$\therefore r_{E.IC} = \sqrt{0.3^2 + 0.35^2} = 0.46\ m$

$v_C = \omega_{CD} \cdot r_{CD} = \omega_{BC} \cdot r_{C.IC}$

$\therefore \omega_{BC} = 6 * \left(\dfrac{0.6}{0.35}\right) = 10.3\ rad/s$

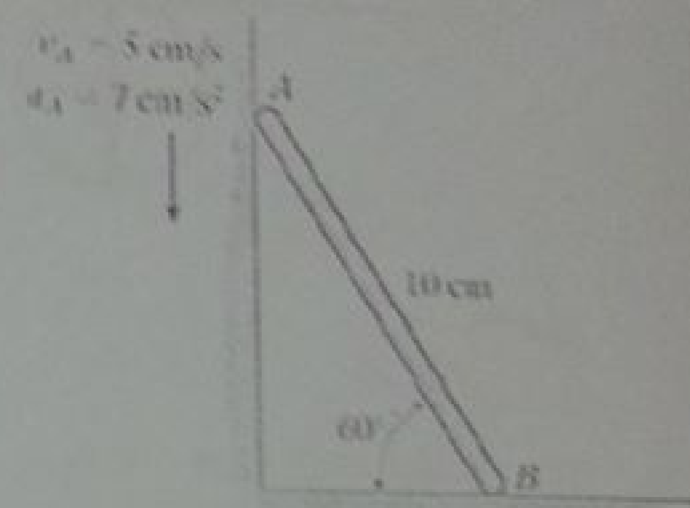
$\therefore v_E = \omega_{BC} \cdot r_{E.IC} = 10.3 * 0.46 = 4.73\ m/s$

$v_B = \omega_{BC} \cdot r_{B.IC} = \omega_{AB} \cdot r_{AB} \quad \therefore \omega_{AB} = 10.3 * \left(\dfrac{0.7}{1.2}\right) = 6\ rad/s$



Velocity and Acceleration Problems

1- At a given instant the top end A of the bar has the velocity and acceleration shown. Determine the acceleration of bottom B and bar's angular acceleration at this instant.



Solution

Givens:

$l = 10\ cm \quad , \quad \theta = 60^o ,$

$v_A = 5\ cm/s \downarrow \quad \& \quad a_A = 7\ cm/s^2 \downarrow$

Req.: $\quad a_B = ?? \quad \& \quad \alpha_{AB} = \alpha_{BA} = ??$ $\quad$ of bar AB

Velocity analysis:

$\overrightarrow{v_B} = \overrightarrow{v_A} + \overrightarrow{\omega_{BA}}\ X\ \overrightarrow{r_{BA}}$

$v_B\ \vec{i} = -5\ \vec{j} + \omega_{BA}\ \vec{k}\ X\ (10\cos 60\ \vec{i} - 10\sin 60\ \vec{j})$

$v_B\ \vec{i} = -5\ \vec{j} + 5\ \omega_{BA}\ \vec{j} + 5\sqrt{3}\ \omega_{BA}\ \vec{i}$

$\vec{j}$ - equation $\quad 0 = -5 + 5\ \omega_{BA} \quad \Rightarrow \quad \therefore \omega_{BA} = 1\ rad/s$ ↶ c.c.w

$\vec{i}$ - equation $\quad v_B = 5\sqrt{3}\ \omega_{BA} \quad \Rightarrow \quad \therefore v_B = 8.66\ m/s \rightarrow$

Acceleration analysis:

$\overrightarrow{a_B} = \overrightarrow{a_A} + [\overrightarrow{\alpha_{BA}}\ X\ \overrightarrow{r_{BA}} - \omega_{BA}{}^2 * \overrightarrow{r_{BA}}]$

$a_B\ \vec{i} = -7\ \vec{j} + \alpha_{BA}\ \vec{k}\ X\ (10\cos 60\ \vec{i} - 10\sin 60\ \vec{j})$

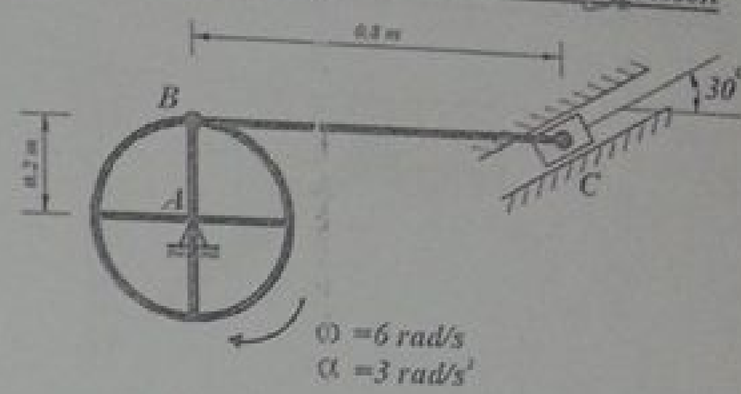
$\qquad - (1)^2 * (10\cos 60\ \vec{i} - 10\sin 60\ \vec{j})$

$a_B\ \vec{i} = -7\ \vec{j} + (5\ \alpha_{BA}\ \vec{j} + 5\sqrt{3}\ \alpha_{BA}\ \vec{i}) - (5\ \vec{i} - 5\sqrt{3}\ \vec{j})$

$\vec{j}$ - equation $\quad 0 = -7 + 5\ \alpha_{BA} + 5\sqrt{3} \quad \Rightarrow \quad \therefore \alpha_{BA} = -0.332\ rad/s^2$ ↷ c.w

$\vec{i}$ - equation $\quad a_B = 5\sqrt{3}\ \alpha_{BA} - 5 \quad \Rightarrow \quad \therefore a_B = -7.87\ m/s^2 \leftarrow$

2-At the instant shown, wheel A rotates with an angular velocity $\omega = 6\, rad/s$ and an angular acceleration $\alpha = 3\, rad/s^2$. Determine the angular acceleration of link BC and the acceleration of point C.



Solution

Givens:

$l_{BC} = 0.8\, m$, $r_{AB} = 0.2\, m$, $\omega_{AB} = 6\, rad/s$ c.w & $\alpha_{AB} = 3\, rad/s^2$ c.w

Req.: $\alpha_{BC} = ??$ & $a_B = ??$ of link BC

Velocity analysis:

$\overrightarrow{v_C} = \overrightarrow{v_B} + \overrightarrow{\omega_{CB}}\, X\, \overrightarrow{r_{CB}}$

where, $\overrightarrow{v_B} = \overrightarrow{\omega_{BA}}\, X\, \overrightarrow{r_{BA}} = -6\, \vec{k}\, X\, (0.2\, \vec{j}) = 1.2\, \vec{i}$

$(v_C \cos 30)\, \vec{i} + (v_C \sin 30)\, \vec{j} = 1.2\, \vec{i} + \omega_{CB}\, \vec{k}\, X\, (0.8\, \vec{i})$

$0.5\sqrt{3}\, v_C\, \vec{i} + 0.5\, v_C\, \vec{j} = 1.2\, \vec{i} + 0.8\, \omega_{CB}\, \vec{j}$

i - equation $0.5\sqrt{3}\, v_C = 1.2 \Rightarrow \therefore v_C = 1.38\, m/s$ ↗

j - equation $0.5\, v_C = 0.8\, \omega_{CB} \Rightarrow \therefore \omega_{CB} = 0.87\, rad/s$ ↺ c.c.w

Acceleration analysis:

$\overrightarrow{a_C} = \overrightarrow{a_B} + [\overrightarrow{\alpha_{CB}}\, X\, \overrightarrow{r_{CB}} - \omega_{CB}^{\ 2} * \overrightarrow{r_{CB}}]$

where, $\overrightarrow{a_B} = [\overrightarrow{\alpha_{BA}}\, X\, \overrightarrow{r_{BA}} - \omega_{BA}^{\ 2} * \overrightarrow{r_{BA}}] = -3\, \vec{k}\, X\, (0.2\, \vec{j}) - (6)^2 * (0.2\, \vec{j})$

$= 0.6\, \vec{i} - 7.2\, \vec{j}$

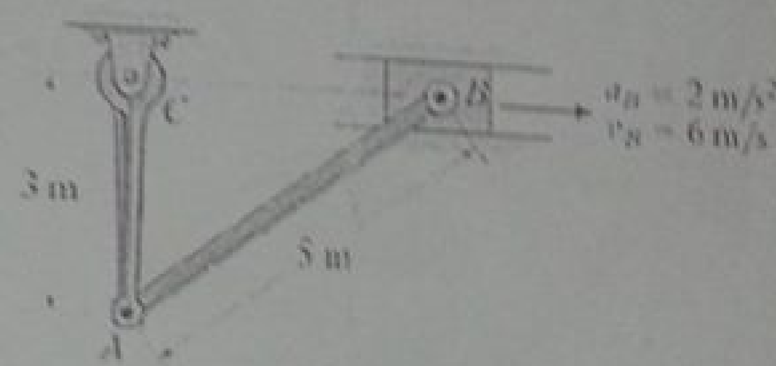
$0.5\sqrt{3}\, a_C\, \vec{i} + 0.5\, a_C\, \vec{j} = 0.6\, \vec{i} - 7.2\, \vec{j} + \alpha_{CB}\, \vec{k}\, X\, (0.8\, \vec{i}) - (0.87)^2 * (0.8\, \vec{i})$

$0.5\sqrt{3}\, a_C\, \vec{i} + 0.5\, a_C\, \vec{j} = 0.6\, \vec{i} - 7.2\, \vec{j} + 0.8\, \alpha_{CB}\, \vec{j} - 0.6\, \vec{i}$

i - equation $0.5\sqrt{3}\, a_C = 0.6 - 0.6 \Rightarrow \therefore a_C = 0$ i.e. const. velocity

j - equation $0.5\, a_C = -7.2 + 0.8\, \alpha_{CB} \Rightarrow \therefore \alpha_{CB} = 9\, rad/s^2$ ↺ c.c.w

3- The slider block B is moving to the right with acceleration $a_B = 2\ m/s^2$. At the instant shown, its velocity is $v_B = 2\ m\ rad/s$ Determine the angular acceleration of link AB and the acceleration of point A at this instant.



Solution

Givens:

$l_{AC} = 3\ m$, $l_{AB} = 5\ m$, $v_B = 6\ m/s\ \rightarrow$ & $a_B = 2\ m/s^2\ \rightarrow$

Req.: $\alpha_{AB} = ??$ & $a_A = ??$ of link AB

Velocity analysis:

$\overrightarrow{v_B} = \overrightarrow{v_A} + \overrightarrow{\omega_{BA}}\ X\ \overrightarrow{r_{BA}}$

where, $\overrightarrow{v_A} = \overrightarrow{\omega_{AC}}\ X\ \overrightarrow{r_{AC}} = \omega_{AC}\ \vec{k}\ X\ (-3\ \vec{j}) = 3\omega_{AC}\ \vec{i}$

$v_B\ \vec{i} = 3\ \omega_{AC}\ \vec{i} + \omega_{BA}\ \vec{k}\ X\ (4\ \vec{i} + 3\ \vec{j})$

$6\ \vec{i} = 3\ \omega_{AC}\ \vec{i} + 4\ \omega_{BA}\ \vec{j} - 3\ \omega_{BA}\ \vec{i}$

j - equation $\quad 0 = 4\ \omega_{BA} \quad \Rightarrow \quad \therefore \omega_{BA} = 0$ *i.e. link AB in translation*

i - equation $\quad 6 = 3\ \omega_{AC} - 3\ \omega_{BA} \quad \Rightarrow \quad \therefore \omega_{AC} = 2\ rad/s$ c.c.w

Acceleration analysis:

$\overrightarrow{a_B} = \overrightarrow{a_A} + [\overrightarrow{\alpha_{BA}}\ X\ \overrightarrow{r_{BA}} - \omega_{BA}{}^2 * \overrightarrow{r_{BA}}]$

where, $\overrightarrow{a_A} = [\overrightarrow{\alpha_{AC}}\ X\ \overrightarrow{r_{AC}} - \omega_{AO}{}^2 * \overrightarrow{r_{AC}}] = \alpha_{AC}\ \vec{k}\ X\ (-3\ \vec{j}) - (2)^2 * (-3\ \vec{j})$

$= 3\ \alpha_{AC}\ \vec{i} + 12\ \vec{j}$

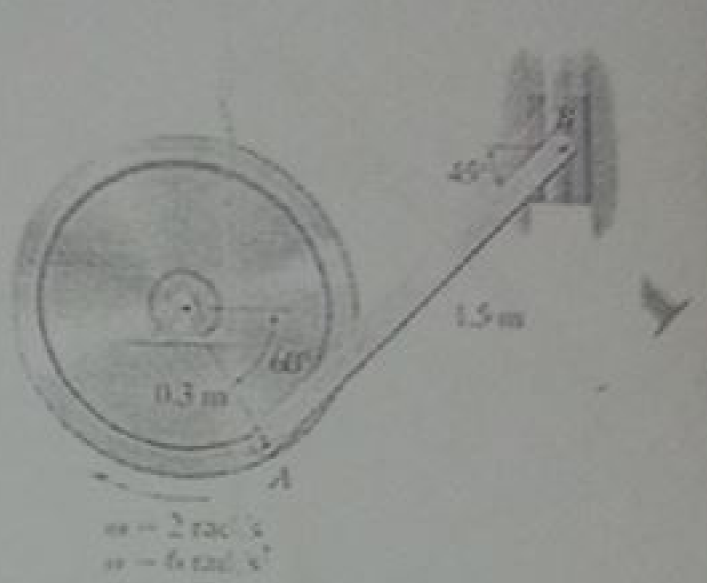
$a_B\ \vec{i} = (3\ \alpha_{AC}\ \vec{i} + 12\ \vec{j}) + \alpha_{BA}\ \vec{k}\ X\ (4\ \vec{i} + 3\ \vec{j}) - (0)^2 * (4\ \vec{i} + 3\ \vec{j})$

$2\ \vec{i} = (3\ \alpha_{AC}\ \vec{i} + 12\ \vec{j}) + (4\ \alpha_{BA}\ \vec{j} - 3\ \alpha_{BA}\ \vec{i}) - 0$

j - equation $\quad 0 = 12 + 4\ \alpha_{BA} \quad \Rightarrow \quad \therefore \alpha_{BA} = -3\ rad/s^2$ c.w

i - equation $\quad 2 = 3\ \alpha_{AC} - 3\ \alpha_{BA} \quad \Rightarrow \quad \therefore \alpha_{AC} = -2.33\ rad/s^2$ c.w

4- At a given instant the wheel is rotating with the angular velocity and angular acceleration shown. Determine the acceleration of block B at this instant.



Givens:

Solution

$l_{AB} = 1.5\ m$, $r_{AO} = 0.3\ m$,

$\omega_{AO} = 2\ rad/s$ c.w & $\alpha_{AO} = 6\ rad/s^2$ c.w

Req.: $a_B = ??$ of block B

Velocity analysis:

$\overrightarrow{v_B} = \overrightarrow{v_A} + \overrightarrow{\omega_{BA}} \times \overrightarrow{r_{BA}}$

where, $\overrightarrow{v_A} = \overrightarrow{\omega_{AO}} \times \overrightarrow{r_{AO}} = -2\ \vec{k} \times (0.3\cos 60\ \vec{i} - 0.3\sin 60\ \vec{j})$

$= -0.3\ \vec{j} - 0.3\sqrt{3}\ \vec{i}$

$v_B\ \vec{j} = (-0.3\ \vec{j} - 0.3\sqrt{3}\ \vec{i}) + \omega_{BA}\ \vec{k} \times (1.5\cos 45\ \vec{i} + 1.5\sin 45\ \vec{j})$

$v_B\ \vec{j} = (-0.3\ \vec{j} - 0.3\sqrt{3}\ \vec{i}) + (0.75\sqrt{2}\ \omega_{BA}\ \vec{j} - 0.75\sqrt{2}\ \omega_{BA}\ \vec{i})$

i - equation $\quad 0 = -0.3\sqrt{3} - 0.75\sqrt{2}\ \omega_{BA} \Rightarrow \therefore \omega_{BA} = -0.49\ rad/s$ c.w

j - equation $\quad v_B = -0.3 + 0.75\sqrt{2}\ \omega_{BA} \Rightarrow \therefore v_B = -0.82\ m/s\ \downarrow$

Acceleration analysis:

$\overrightarrow{a_B} = \overrightarrow{a_A} + [\overrightarrow{\alpha_{BA}} \times \overrightarrow{r_{BA}} - \omega_{BA}^2 * \overrightarrow{r_{BA}}]$

where, $\overrightarrow{a_A} = [\overrightarrow{\alpha_{AO}} \times \overrightarrow{r_{AO}} - \omega_{AO}^2 * \overrightarrow{r_{AO}}] = -6\ \vec{k} \times (0.3\cos 60\ \vec{i} - 0.3\sin 60\ \vec{j})$

$-(2)^2 * (0.3\cos 60\ \vec{i} - 0.3\sin 60\ \vec{j})$

$= (-0.9\ \vec{j} - 0.9\sqrt{3}\ \vec{i}) - (0.6\ \vec{i} - 0.6\sqrt{3}\ \vec{j}) = -2.16\ \vec{i} + 0.14\ \vec{j}$

$a_B\ \vec{j} = (-2.16\ \vec{i} + 0.14\ \vec{j}) + \alpha_{BA}\ \vec{k} \times (1.5\cos 45\ \vec{i} + 1.5\sin 45\ \vec{j})$

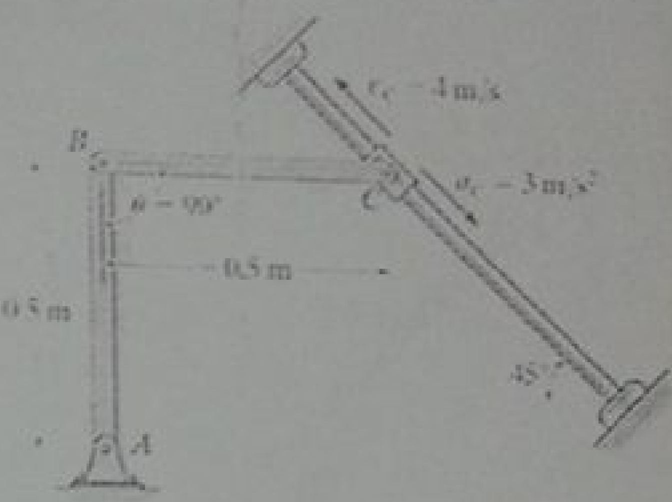
$-(0.49)^2 * (1.5\cos 45\ \vec{i} + 1.5\sin 45\ \vec{j})$

$a_B\ \vec{j} = (-2.16\ \vec{i} + 0.14\ \vec{j}) + (0.75\sqrt{2}\ \alpha_{BA}\ \vec{j} - 0.75\sqrt{2}\ \alpha_{BA}\vec{i}) - (0.18\sqrt{2}\ \vec{i} + 0.18\sqrt{2}\ \vec{j})$

i - equation $\quad 0 = -2.16 - 0.75\sqrt{2}\ \alpha_{BA} - 0.18\sqrt{2} \Rightarrow \therefore \alpha_{BA} = -2.27\ r/s^2$ c.w

j - equation $\quad a_B = 0.14 + 0.75\sqrt{2}\ \alpha_{BA} - 0.18\sqrt{2} \Rightarrow \therefore a_B = -2.53\ m/s^2\ \downarrow$

5- Determine the angular acceleration of the link AB at the instant shown if the collar C has velocity $v_c = 4\ m/s$, and deceleration $a_c = 3\ m/s^2$ as shown.



Solution

Givens:

$l_{AB} = 0.5\ m$, $l_{BC} = 0.5\ m$,

$v_C = 4\ m/s$ & $a_C = -3\ m/s^2$

Req.: $\alpha_{AB} = ??$ of link AB

Velocity analysis:

$\overrightarrow{v_C} = \overrightarrow{v_B} + \overrightarrow{\omega_{CB}} X \overrightarrow{r_{CB}}$

where, $\overrightarrow{v_B} = \overrightarrow{\omega_{BA}} X \overrightarrow{r_{BA}} = \omega_{BA}\ \vec{k}\ X\ (0.5\ \vec{j}) = -0.5\ \omega_{BA}\ \vec{i}$

$-(v_C \cos 45)\ \vec{i} + (v_C \sin 45)\ \vec{j} = -0.5\ \omega_{BA}\ \vec{i} + \omega_{CB}\ \vec{k}\ X\ (0.5\ \vec{i})$

$-2\sqrt{2}\ \vec{i} + 2\sqrt{2}\ \vec{j} = -0.5\ \omega_{BA}\ \vec{i} + 0.5\ \omega_{CB}\ \vec{j}$

i - equation $-2\sqrt{2} = -0.5\ \omega_{BA}$ $\Rightarrow$ $\therefore \omega_{BA} = 4\sqrt{2}\ rad/s$ c.c.w

j - equation $2\sqrt{2} = 0.5\ \omega_{CB}$ $\Rightarrow$ $\therefore \omega_{CB} = 4\sqrt{2}\ rad/s$ c.c.w

Acceleration analysis:

$\overrightarrow{a_C} = \overrightarrow{a_B} + [\overrightarrow{\alpha_{CB}} X \overrightarrow{r_{CB}} - \omega_{CB}^2 * \overrightarrow{r_{CB}}]$

where, $\overrightarrow{a_B} = [\overrightarrow{\alpha_{BA}} X \overrightarrow{r_{BA}} - \omega_{BA}^2 * \overrightarrow{r_{BA}}] = \alpha_{BA}\ \vec{k}\ X\ (0.5\ \vec{j}) - (4\sqrt{2})^2 * (0.5\ \vec{j})$

$= -0.5\ \alpha_{BA}\ \vec{i} - 16\ \vec{j}$

$(a_C \cos 45)\ \vec{i} - (a_C \sin 45)\ \vec{j} = (-0.5\ \alpha_{BA}\ \vec{i} - 16\ \vec{j}) + \alpha_{CB}\ \vec{k}\ X\ (0.5\ \vec{i})$

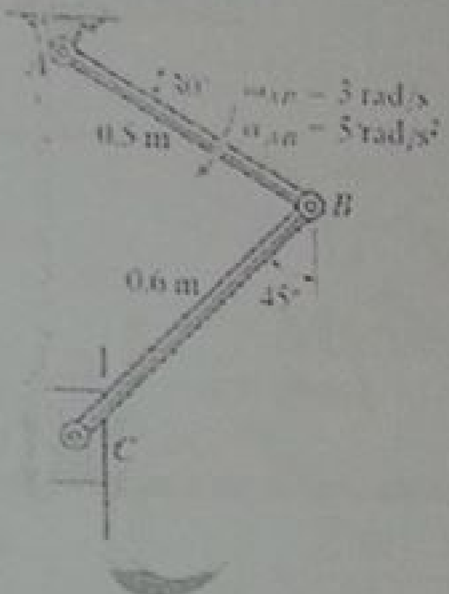
$- (4\sqrt{2})^2 * (0.5\ \vec{i})$

$1.5\sqrt{2}\ \vec{i} - 1.5\sqrt{2}\ \vec{j} = (-0.5\ \alpha_{BA}\ \vec{i} - 16\ \vec{j}) + (0.5\ \alpha_{CB}\ \vec{j}) - (16\ \vec{i})$

i - equation $1.5\sqrt{2} = -0.5\ \alpha_{BA} - 16$ $\Rightarrow$ $\therefore \alpha_{BA} = -36.24\ rad/s^2$ c.w

j - equation $-1.5\sqrt{2} = -16 + 0.5\ \alpha_{CB}$ $\Rightarrow$ $\therefore \alpha_{CB} = 27.76\ rad/s^2$ c.c.w

6- Rod AB has angular motion shown. Determine the acceleration of collar C at this instant.



Solution

Givens:

$l_{AB} = 0.5\ m$, $l_{BC} = 0.6\ m$,

$\omega_{AB} = 3\ rad/s$ c.w & $\alpha_{AB} = 5\ rad/s^2$ c.w

Req.: $a_C = ??$ of collar C

Velocity analysis:

$\overrightarrow{v_C} = \overrightarrow{v_B} + \overrightarrow{\omega_{CB}}\ X\ \overrightarrow{r_{CB}}$

where, $\overrightarrow{v_B} = \overrightarrow{\omega_{BA}}\ X\ \overrightarrow{r_{BA}} = -3\ \vec{k}\ X\ (0.5\cos 30\ \vec{i} - 0.3\sin 30\ \vec{j})$

$= -0.75\sqrt{3}\ \vec{j} - 0.75\ \vec{i}$

$v_C\,\vec{j} = (-0.75\sqrt{3}\ \vec{j} - 0.75\ \vec{i}) + \omega_{CB}\ \vec{k}\ X\ (-0.6\sin 45\ \vec{i} - 0.6\cos 45\ \vec{j})$

$v_C\,\vec{j} = (-0.75\sqrt{3}\ \vec{j} - 0.75\ \vec{i}) + (-0.3\sqrt{2}\,\omega_{CB}\ \vec{j} + 0.3\sqrt{2}\,\omega_{CB}\ \vec{i})$

i - equation $\quad 0 = -0.75 + 0.3\sqrt{2}\,\omega_{CB} \quad \Rightarrow \quad \therefore \omega_{CB} = 1.77\ rad/s$ ↶ c.c.w

j - equation $\quad v_C = -0.75\sqrt{3} - 0.3\sqrt{2}\,\omega_{CB} \quad \Rightarrow \quad \therefore v_C = -2.05\ m/s$ ↓

Acceleration analysis:

$\overrightarrow{a_C} = \overrightarrow{a_B} + [\overrightarrow{\alpha_{CB}}\ X\ \overrightarrow{r_{CB}} - \omega_{CB}^2 * \overrightarrow{r_{CB}}]$

where, $\overrightarrow{a_B} = [\overrightarrow{\alpha_{BA}}\ X\ \overrightarrow{r_{BA}} - \omega_{BA}^2 * \overrightarrow{r_{BA}}] = -5\ \vec{k}\ X\ (0.5\cos 30\ \vec{i} - 0.3\sin 30\ \vec{j})$

$-(3)^2 * (0.5\cos 30\ \vec{i} - 0.3\sin 30\ \vec{j})$

$= (-1.25\sqrt{3}\ \vec{j} - 1.25\ \vec{i}) - (2.25\sqrt{3}\ \vec{i} - 2.25\ \vec{j}) = -5.15\ \vec{i} + 0.085\ \vec{j}$

$a_C\,\vec{j} = (-5.15\ \vec{i} + 0.085\ \vec{j}) + \alpha_{CB}\ \vec{k}\ X\ (-0.6\sin 45\ \vec{i} - 0.6\cos 45\ \vec{j})$

$- (1.77)^2 * (-0.6\sin 45\ \vec{i} - 0.6\cos 45\ \vec{j})$

$a_C\,\vec{j} = (-5.15\ \vec{i} + 0.085\ \vec{j}) + (-0.3\sqrt{2}\,\alpha_{CB}\ \vec{j} + 0.3\sqrt{2}\,\alpha_{CB}\vec{i}) - (-0.94\sqrt{2}\ \vec{i} - 0.94\sqrt{2}\ \vec{j})$

i - equation $\quad 0 = -5.15 + 0.3\sqrt{2}\,\alpha_{CB} + 0.94\sqrt{2} \quad \Rightarrow \quad \therefore \alpha_{CB} = 9\ r/s^2$ ↶ c.c.w

j - equation $\quad a_C = 0.085 - 0.3\sqrt{2}\,\alpha_{CB} + 0.94\sqrt{2} \quad \Rightarrow \quad \therefore a_C = -2.41\ m/s^2$ ↓

Ch. (2) - Centroids and Center of Mass

Wires (Lines)	Surfaces (Areas)	Solids (Volumes)	Mass
$X_c = \frac{\sum L_i . x_i}{\sum L_i}$	$X_c = \frac{\sum A_i . x_i}{\sum A_i}$	$X_c = \frac{\sum V_i . x_i}{\sum V_i}$	$X_c = \frac{\sum m_i . x_i}{\sum m_i}$
$Y_c = \frac{\sum L_i . y_i}{\sum L_i}$	$Y_c = \frac{\sum A_i . y_i}{\sum A_i}$	$Y_c = \frac{\sum V_i . y_i}{\sum V_i}$	$Y_c = \frac{\sum m_i . y_i}{\sum m_i}$
$Z_c = \frac{\sum L_i . z_i}{\sum L_i}$	$Z_c = \frac{\sum A_i . z_i}{\sum A_i}$	$Z_c = \frac{\sum V_i . z_i}{\sum V_i}$	$Z_c = \frac{\sum m_i . z_i}{\sum m_i}$

$\rho = Constant$ (Wires, Surfaces, Solids)

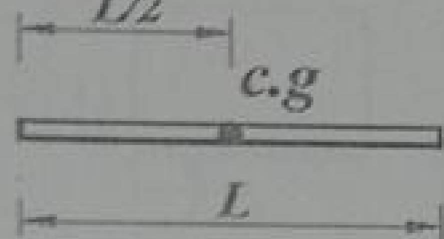
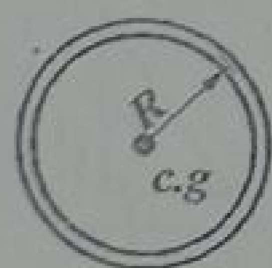
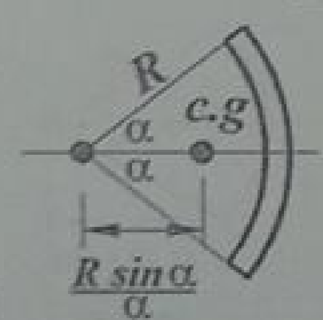
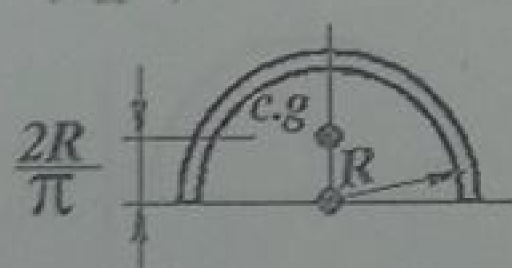
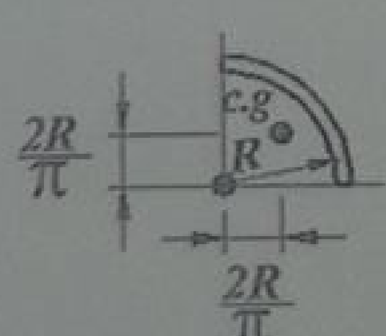
$\rho \neq Const.$ (Mass)

$m = \rho * L = \quad kg$ for Line, where $\Rightarrow \rho = \quad kg/m$

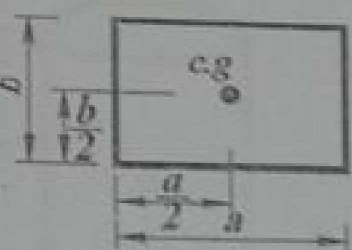
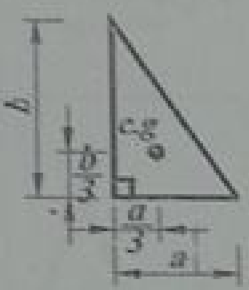
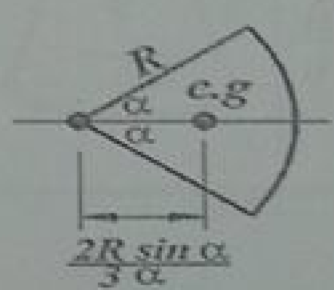
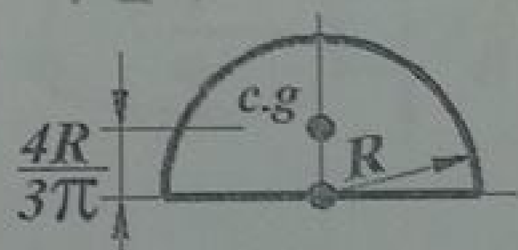
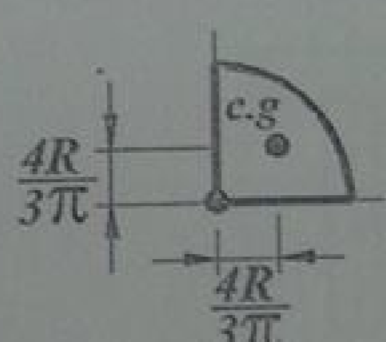
$m = \rho * A = \quad kg$ for Area, where $\Rightarrow \rho = \quad kg/m^2$

$m = \rho * V = \quad kg$ for Volume, where $\Rightarrow \rho = \quad kg/m^3$

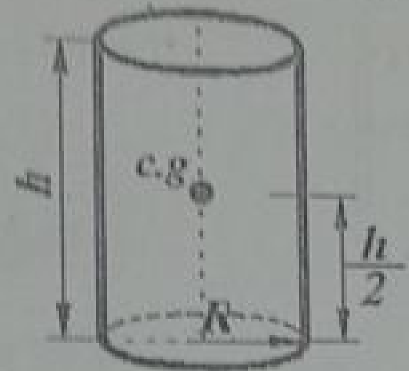
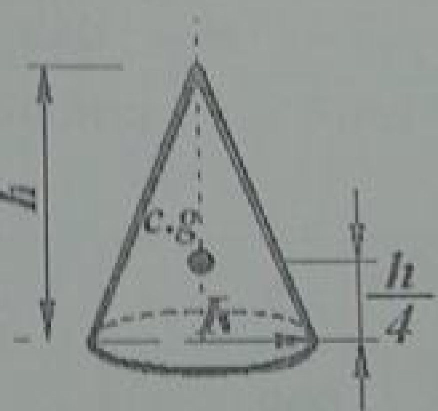
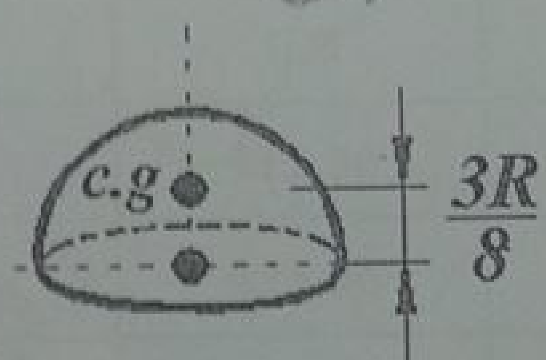
Centroid of lines

Name	Description	Length	Centriod
Straight wire		L	$x_c = \frac{L}{2}$
Ring		$2\pi.R$	——
Arc		$2\alpha.R$	$x_c = \frac{R.\sin\alpha}{\alpha}$
Half ring		$\pi.R$	$y_c = \frac{2R}{\pi}$
Quarter ring		$\frac{\pi R}{2}$	$x_c = \frac{2R}{\pi}$ $y_c = \frac{2R}{\pi}$

Centroid of areas

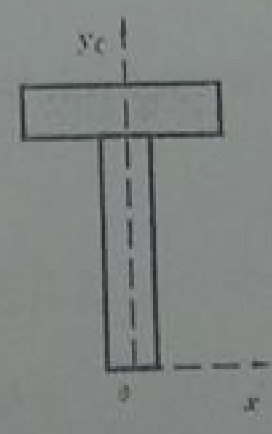
Name	Description	Area	Centriod
Rectangle		$a.b$	$x_c = \frac{a}{2}$ $y_c = \frac{b}{2}$
Triangle		$\frac{a.b}{2}$	$x_c = \frac{a}{3}$ $y_c = \frac{b}{3}$
Sector		$\alpha.R^2$	$x_c = \frac{2R.\sin\alpha}{3\alpha}$
Half Disk		$\frac{\pi.R^2}{2}$	$y_c = \frac{4R}{3\pi}$
Quarter Disk		$\frac{\pi.R^2}{4}$	$x_c = \frac{4R}{3\pi}$ $y_c = \frac{4R}{3\pi}$

Centroid of volumes

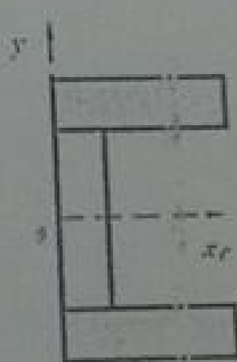
Name	Description	Volume	Centriod
Cylinder		$\pi . R^2 . h$	$z_c = \frac{h}{2}$
Right Cone		$\frac{\pi . R^2 . h}{3}$	$z_c = \frac{h}{4}$
Half Sphere		$\frac{4\pi . R^3}{6}$	$z_c = \frac{3}{8} R$

if the symmetry exist;

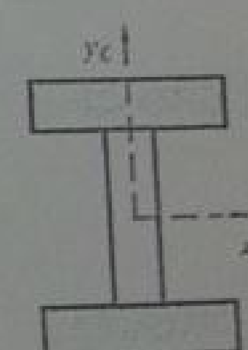
it should considered as:



If Symmetery about Y axis $X_c = 0$



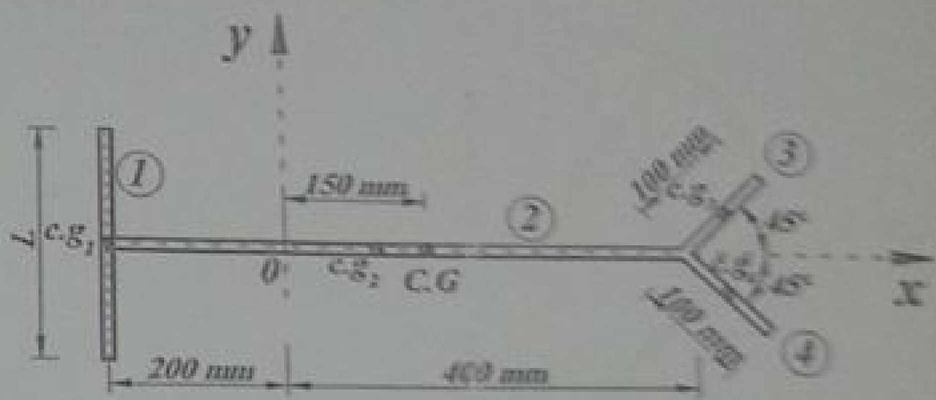
If Symmetery about X axis $Y_c = 0$



If Symmetery about X & Y axes $X_c = Y_c = 0$

Centeroids Problems

1-Determine the length L so that the centroid of the given bent wire is located at point C.



Solution

* لاحظ التماثل حول المحور الأفقى.

$from\ symmetry\ about\ Y\ axis \quad \Rightarrow \quad \therefore Y_c = 0$

$Ginen: \quad \Rightarrow \quad X_c = 150\ mm \qquad Find \quad L_1 = ??\ mm$

#	L_i	x_i	$L_i \cdot x_i$
1	L	-200	-200 L
2	600	100	60,000
3	100	435.36	43,535.5
4	100	435.36	43,535.5
$\sum$	800+L		-200 L+147,071

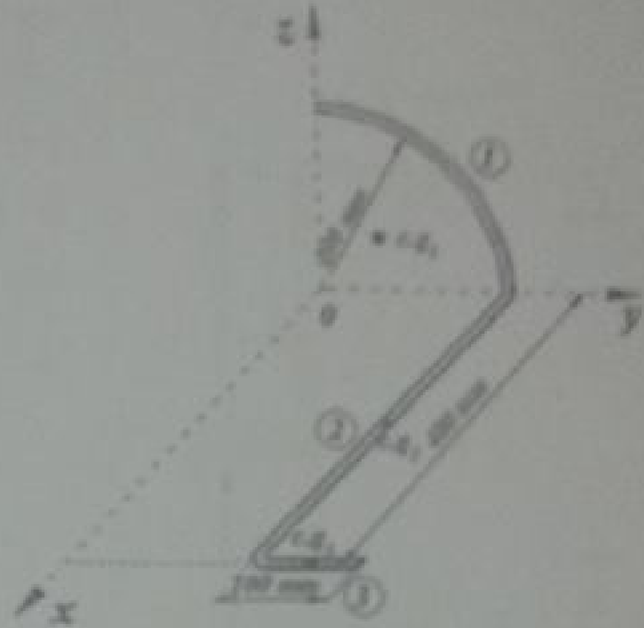
$$X_c = \frac{\sum L_i \cdot x_i}{\sum L_i} = \frac{-200\,L + 147{,}071}{800 + L} = 150\ mm$$

$$-200\,L + 147{,}071 = 120{,}000 + 150\,L$$

$$147{,}071 - 120{,}000 = 150\,L + 200\,L$$

$$\therefore L = \frac{27{,}071}{350} = 77.35\ mm$$

2- Locate the center of mass of the homogenous rod bent into given shape.



Solution

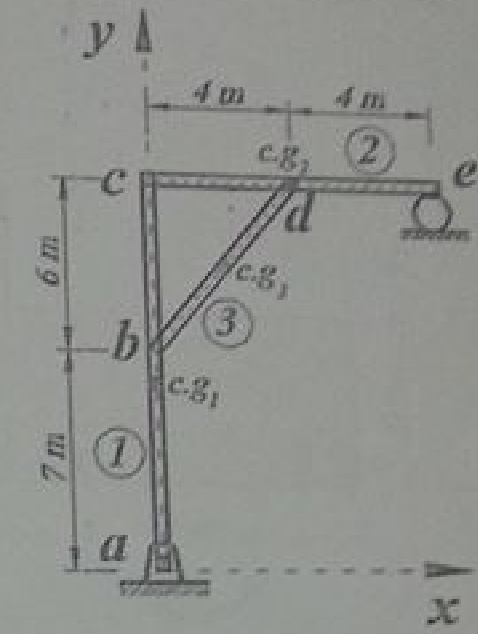
#	l_i	x_i	y_i	z_i	$l_i . x_i$	$l_i . y_i$	$l_i . z_i$
1	314.16	0	127.32	127.32	0	40,000	40,000
2	400	200	200	0	80,000	80,000	0
3	100	400	250	0	40,000	25,000	0
$\sum$	814.16				120,000	145,000	40,000

$$\therefore X_c = \frac{\sum l_i . x_i}{\sum l_i} = \frac{120,000}{814.16} = 147.4\ mm$$

$$\therefore Y_c = \frac{\sum l_i . y_i}{\sum l_i} = \frac{145,000}{814.16} = 178.1\ mm$$

$$\therefore Z_c = \frac{\sum l_i . z_i}{\sum l_i} = \frac{40,000}{814.16} = 49.13\ mm$$

3-Each of the three members of the frame has a mass per unit length 6 kg/m. Locate the location of the center of gravity. Neglect the size of the pins at joints and the thickness of the members. Also calculate the reaction at the pin A and roller B.



Solution

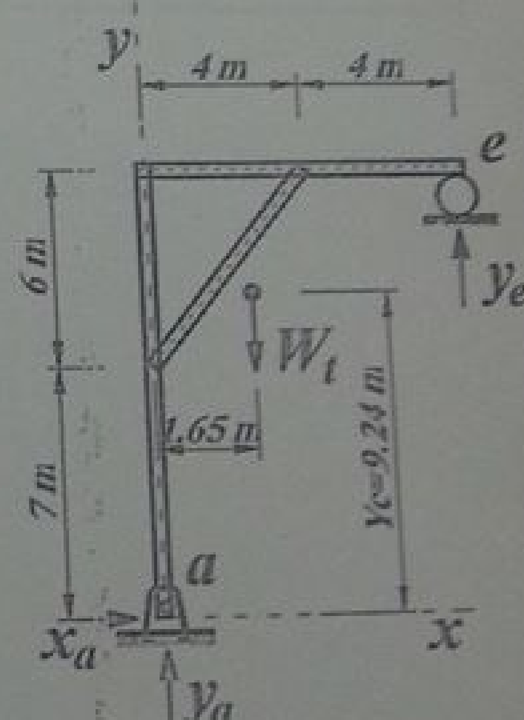
$$m = \rho * L = \ kg$$

#	m_i	x_i	y_i	$m_i . x_i$	$m_i . y_i$
1	78	0	6.5	0	507
2	48	4	13	192	624
3	43.27	2	10	86.54	432.7
$\sum$	169.27			278.54	1,563.7

$$X_c = \frac{\sum m_i . x_i}{\sum m_i} = \frac{278.54}{169.27} = 1.65\ m$$

$$Y_c = \frac{\sum m_i . y_i}{\sum m_i} = \frac{1,563.7}{169.27} = 9.24\ m$$

__For the support reactions:__



$$W_t = \sum m_i * g = 169.27 * 9.81 = 1660.5\ N$$

$$\sum F_x = 0 \quad \Rightarrow \quad \therefore X_a = 0$$

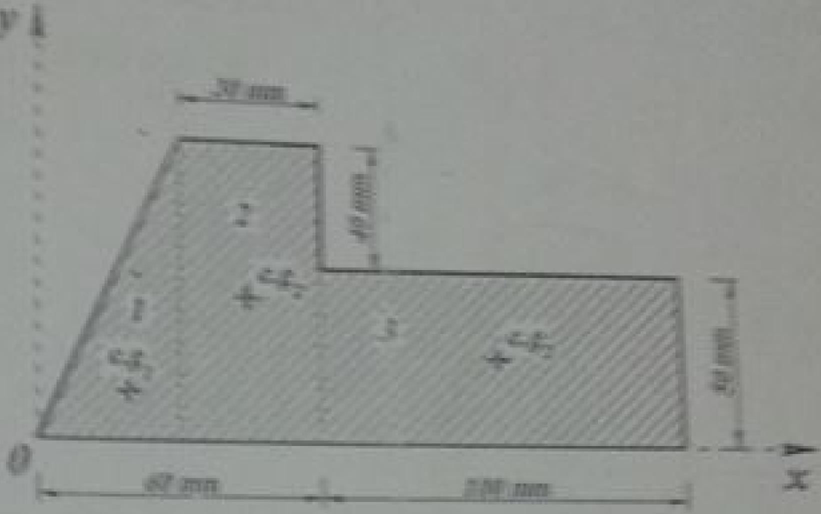
$$\sum M_a = 0$$

$$1660.5 * 1.65 - Y_e * 8 = 0 \quad \Rightarrow \quad \therefore Y_e = 342.5\ N$$

$$\sum F_y = 0$$

$$Y_a + Y_e - 1660.54 = 0 \quad \Rightarrow \quad \therefore Y_a = 1318\ N$$

4-Determine the location X_c and Y_c of the centroid C of the given area.



Solution

* لاحظ ان الاحداثيات كلها مرصودة من نقطة الاصل للمحاور.

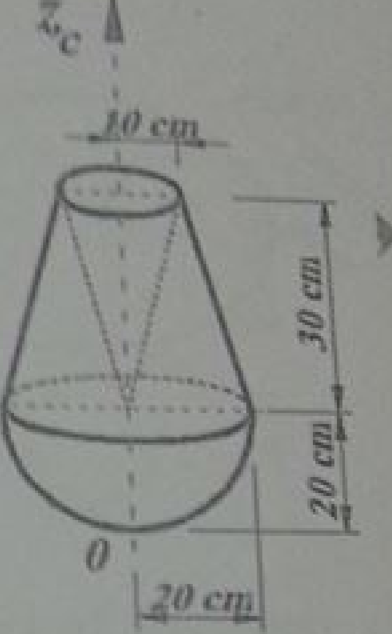
Part	A_i	x_i	y_i	$A_i \cdot x_i$	$A_i \cdot y_i$
1	1,350	20	30	27,000	40,500
2	2,700	45	45	121,500	121,500
3	50,000	110	25	5,500,000	1,250,000
$\sum$	54,050			5,648,500	1,412,000

The coordinates of the centriod are:

$$X_c = \frac{\sum A_i \cdot x_i}{\sum A_i} = \frac{5,648,500}{54,050} = 104.5\ mm$$

$$Y_c = \frac{\sum A_i \cdot y_i}{\sum A_i} = \frac{1,412,000}{54,050} = 26.12\ mm$$

5-Locate the center of mass of the composite assembly shown. The material density is $7850\ kg/m^3$.

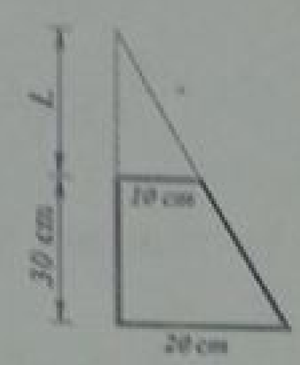


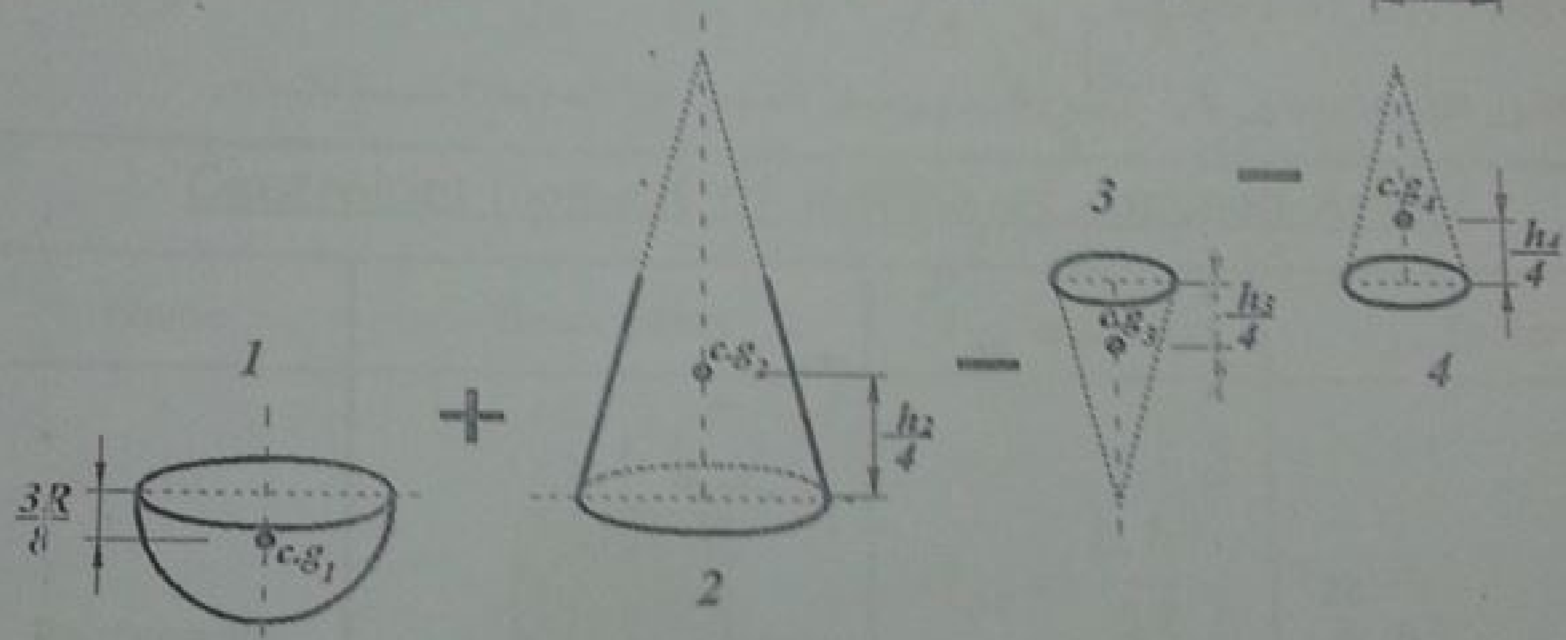
Solution

لإيجاد ارتفاع المخروط العلوى من التشابه

$$\frac{10}{20} = \frac{L}{L+30}$$

$$\therefore L = 30\ cm$$



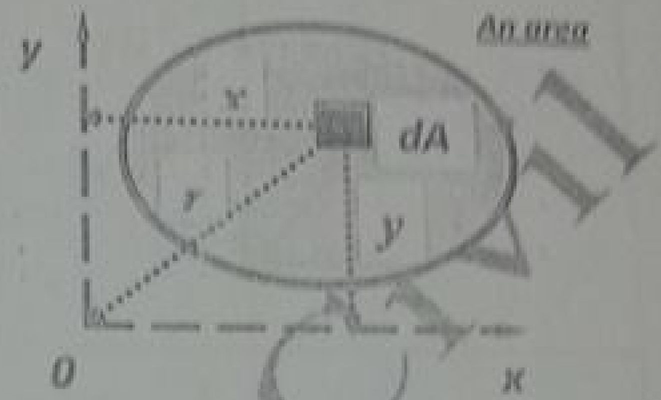


Part	V_i	z_i	$V_i \cdot z_i$
1	16,755.16	12.5	209,439.5
2	25,132.74	35	879,645.9
3	-3,141.6	42.5	-133,518
4	-3,141.6	57.5	-180,642
$\sum$	35,604.7		774,925.4

$$Z_c = \frac{\sum A_i \cdot y_i}{\sum A_i} = \frac{774{,}925.4}{35{,}604.7} = 21.76\ cm$$

Ch. (3) (A) - Area Moment of Inertia

$$I_X = \int y^2 . dA$$



$$I_Y = \int x^2 . dA$$

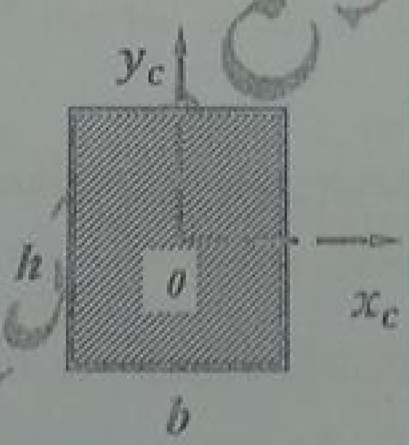
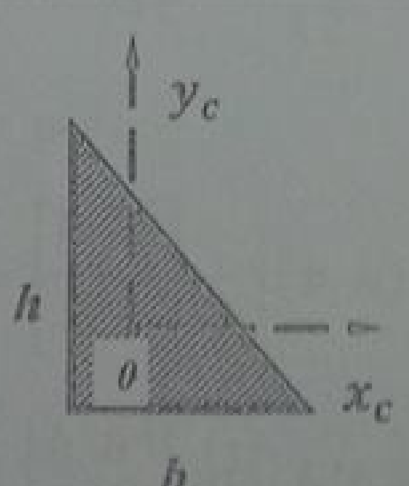
$$I_Z = I_0 = I_P = \int r^2 . dA = \int (x^2 + y^2) . dA$$

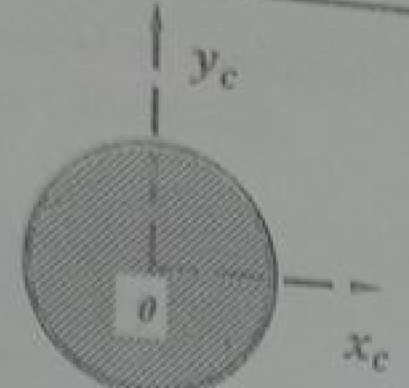
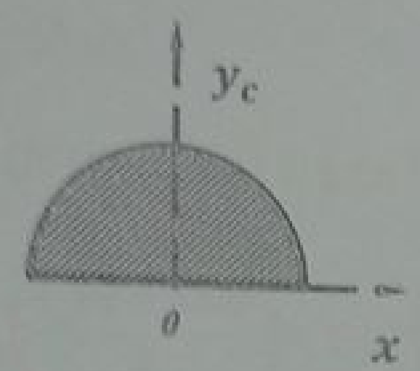
$$= I_X + I_Y$$ ....Polar moment of inertia

$$K_X = \sqrt{\frac{I_X}{A}} \quad , \quad K_Y = \sqrt{\frac{I_Y}{A}} \quad \& \quad K_P = \sqrt{\frac{I_P}{A}}$$ ....Radius of Gyration

عزم القصور الذاتى دائما يكون كمية موجبة ولاتساوى صفر لانة يعبر عن""مقاومة"" الجسم للانحناء.

Centroidal moment of inertia of common Areas

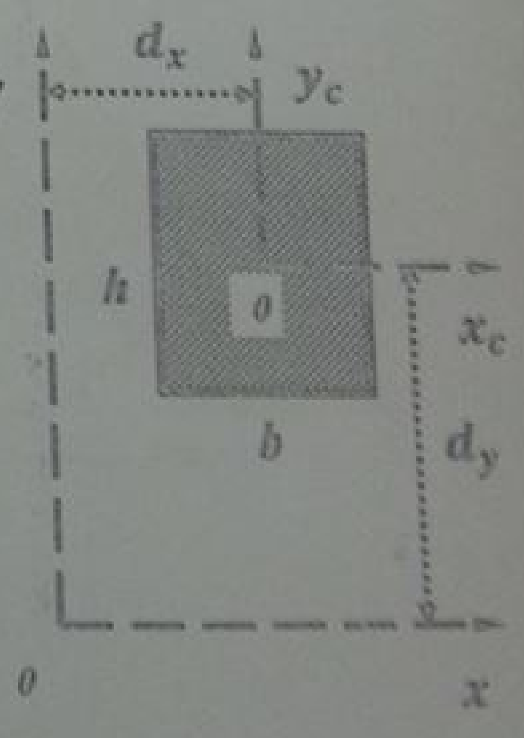
Name	Description	I_{Xc} and I_{Yc}	I_{Zc} (polar)
Rectangular		$I_{Xc} = \frac{b.h^3}{12}$ $I_{Yc} = \frac{h.b^3}{12}$	$I_{Zc} = \frac{b.h^3}{12} + \frac{h.b^3}{12}$
Triangle		$I_{Xc} = \frac{b.h^3}{36}$ $I_{Yc} = \frac{h.b^3}{36}$	$I_{Zc} = \frac{b.h^3}{36} + \frac{h.b^3}{36}$

Name	Description	I_{Xc} and I_{Yc}	I_{Zc}
Circle		$I_{Xc} = \frac{\pi R^4}{4}$ $I_{Yc} = \frac{\pi R^4}{4}$	$I_{Zc} = \frac{\pi R^4}{2}$
Half Circle		$I_X = \frac{\pi R^4}{8}$ $I_{Yc} = \frac{\pi R^4}{8}$	$I_0 = \frac{\pi R^4}{4}$

Parallel axes theorem

To get the moment of inertia about x and y axes in the given area follow this theory.

أقل قيمة لعزم القصور تكون عند المحاور المارة بمركز الثقل.
وكلما بعدت المحاور عن مركز الثقل تزيد هذة الكمية كما يلى:



The Centriodal moment of inertia (I_{Xc} and I_{Yc}) are:

$$I_{Xc} = \frac{b.h^3}{12} \quad , \quad I_{Yc} = \frac{h.b^3}{12}$$

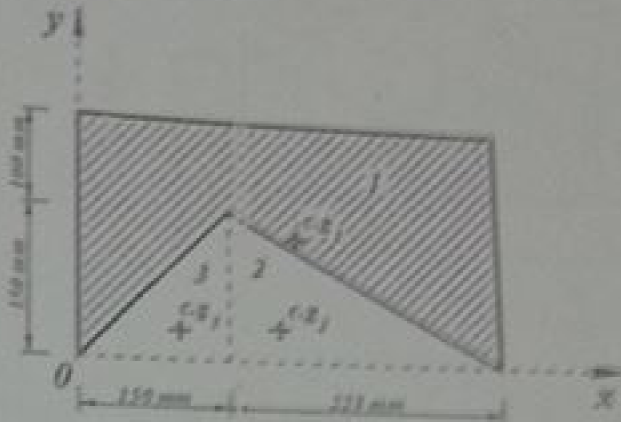
Then, the I_X and I_Y are:

$$I_X = I_{Xc} + A.d_y^{\,2} = \left[\frac{b.h^3}{12}\right] + \left[(b.h) * d_y^{\,2}\right]$$

$$I_Y = I_{Yc} + A.d_x^{\,2} = \left[\frac{h.b^3}{12}\right] + \left[(b.h) * d_x^{\,2}\right]$$

Area Moment of Inertia Problems

1- Calculate the moment of inertia of the shaded area about the x-axis.



Solution

The moment of inertia about X axis:

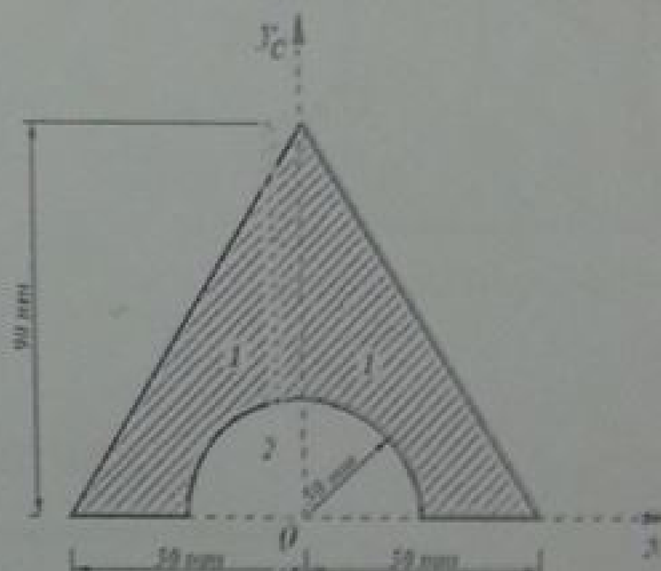
$$I_X = [I_X]_1 - [I_X]_2 - [I_X]_3$$

$$= \left[\frac{375 * 250^3}{12} + (375 * 250) * 125^2\right] - \left[\frac{225 * 150^3}{36} + \left(\frac{225 * 150}{2}\right) * 50^2\right]$$

$$- \left[\frac{150 * 150^3}{36} + \left(\frac{150 * 150}{2}\right) * 50^2\right] = 1.86 * 10^9\ mm^4$$

$$+2 * \left[\frac{159 * 17.6^3}{12} + (159 * 17.6) * 221.2^2\right] = 389.62 * 10^6\ mm^4$$

2- Calculate the moment of inertia of the shaded area about the x-axis.



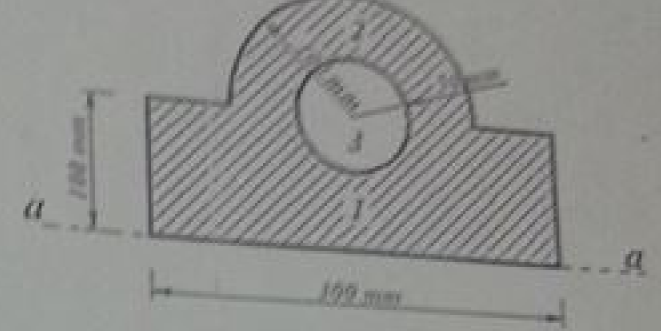
Solution

$$I_X = 2.[I_X]_1 - [I_X]_2$$

$$= 2 * \left[\frac{50 * 90^3}{36} + \left(\frac{50 * 90}{2}\right) * 30^2\right]$$

$$- \left[\frac{\pi * 30^4}{8} + 0\right] = 5.76 * 10^6\ mm^4$$

3- The cross section of a bearing block is shown in the figure by the shaded area. Calculate the moment of inertia of the section about its base a-a.



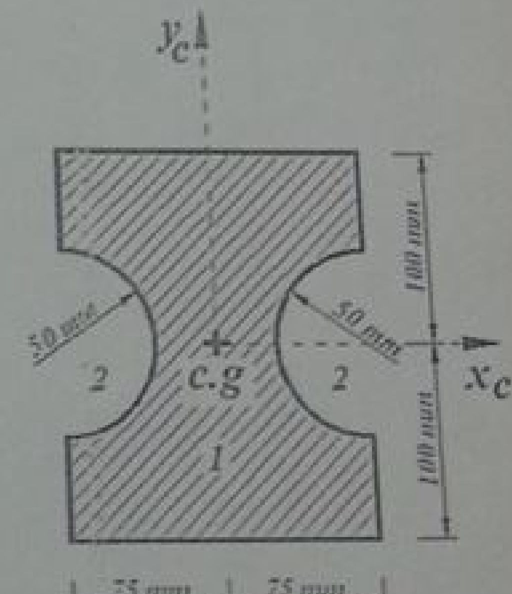
Solution

$$I_{a-a} = [I_X]_1 + [I_X]_2 - [I_X]_3$$

$$= \left[\frac{300 * 100^3}{12} + (300 * 100) * 50^2\right] + \left[\frac{\pi * 100^4}{8} + \left(\frac{\pi * 100^2}{2}\right) * 100^2\right]$$

$$- \left[\frac{\pi * 50^4}{4} + (\pi * 50^2) * 100^2\right] = 212.9 * 10^6\ mm^4$$

4- Calculate the moment of inertia of the shaded area about the centeriodal axes.



Solution

I_{yc} for half circle

$$I_y = I_{yc} + A.d_x^{\,2}$$

$$\frac{\pi * 50^4}{8} = I_{yc} + \left(\frac{\pi * 50^2}{2}\right) \cdot \left(\frac{4 * 50}{3\pi}\right)^2$$

$$I_{yc} = \frac{\pi * 50^4}{8} - \left(\frac{\pi * 50^2}{2}\right) \cdot \left(\frac{4 * 50}{3\pi}\right)^2 = 685{,}981$$

$$I_{Xc} = \left[\frac{150 * 200^3}{12} + 0\right] - 2 * \left[\frac{\pi * 50^4}{8} + 0\right] = 95.1 * 10^6\ mm^4$$

$$I_{Yc} = \left[\frac{200 * 150^3}{12} + 0\right] - 2 * \left[685{,}981 + \left(\frac{\pi * 50^2}{2}\right) * \left(75 - \frac{4 * 50}{3\pi}\right)^2\right]$$

$$= 32.16 * 10^6\ mm^4$$