Permutation

Ex : how many 2 different digits numbers can be formed from 1 , 2 , 3 and 4
Sol : The ways of choice the unit digits equals 4
The ways of choice the tenth digits equals 3
The number of choice for forming 2 different digits number $= 4 \times 3 = 12$

Ex : three tourists arrived at Alex. In which there are 5 hotels. How many way can each one be hosted alone in one of these hotels .
The 1^{st} *tourist choose one of 5 hotels*
The 2^{nd} *tourist choose one of 4 hotels*
The 3^{rd} *tourist choose one of 3 hotels*
The number of different ways for hosting $= 5 \times 4 \times 3 = 60$

The counting principle

if a certain act is formed by M different ways
And a second act is formed N different ways
So can be formed the two acts on succession by $M \times N$ *different ways*

Ex : How many2 different digits numbers can be formed from 1 ,2 ,3 ,4 ,5 and 6 ?
The unit digit can be chosen by 6 different ways and the way of choice the tenth digits equals 5 $\therefore$ *the number of choice for 2 different digits number* $= 6 \times 5 = 30$

Ex : Four tourists arrived at Cairo . In which there are 10 hotels. How many ways can each one are hosted alone in one of these hotels.

The 1^{st} *tourists choose one of 10 hotels, while the* 2^{nd} *has only 9 hotels , the* 3^{rd} *has 8 and the* 4^{th} *has 7* $\therefore$ *the number of different ways for hosting equals*

$$10 \times 9 \times 8 \times 7 =$$

Definition

Given a set of elements
Each arrangement can be formed by taking some or all of these elements in a definite
Is called permutations

N . T : if the given elements n , the taking elements r where $n \geq r$, $n \in \mathbb{Z}^+$, so the number of the different arrangement denoted to it by ${}^{n}P_{r}$
Where n is called the flag , r is called guide .

Remark : can be written each of the pervious ex . by using the symbol for permutations

1) ${}^{6}P_{2}$ 2) ${}^{10}P_{4}$

1^{st} number of 2 different digits number ${}^{6}P_{2} = 6 \times 5 = 30$

2^{nd} Formed permutations of 10 hotels taken 4 at time equals ${}^{10}P_{4} = 10 \times 9 \times 8 \times 7$

Rule : ${}^{n}P_{r} = n(n-1)(n-2)(n-3)\ldots\ldots(n-r+1)$, $n, r \in \mathbb{Z}^+$, $n \geq r$

N . B ${}^{n}P_{0} = 1$

Ex : Write the value of

${}^{4}P_{2}$

${}^{5}P_{2}$

${}^{10}P_{5}$

Ex : Find the value of x in each of the following

${}^{x}P_{4} = 1680$

${}^{8}P_{x} = 6720$

Ex; If ${}^{x+2}P_{3} = 110\,x$; so find the value of ${}^{x}P_{2}$.

Write down the value of each of; $^{3}P_{3}$, $^{5}P_{5}$, $^{7}P_{7}$.

N. B : $^{n}P_{n}$ *Factorial and denoted to it by* $\underline{|n}$.

$$\boxed{\underline{|n} = n\,(n-1)\,(n-2)\,(n-3)\,(n-4)\,........4\times3\times2\times1}$$

Ex: Find $\underline{|4}$ $\underline{|10}$ $\dfrac{\underline{|5}}{\underline{|4}}$

$\dfrac{\underline{|13}}{\underline{|12}}$ deduce $\dfrac{\underline{|n}}{\underline{|n-1}}$

corollary : $\underline{|n} = n\,\underline{|n-1}$.

$\underline{|5} = 5\underline{|4}$

Ex : prove that $\dfrac{\underline{|n+1}}{\underline{|n-1}} - \dfrac{\underline{|n}}{\underline{|n-2}} = 2n$

Ex : If $\underline{|2n-1} = 5040$, so find $^{n}p_{2}$

EX: If $\dfrac{\underline{|n-2}}{\underline{|n-4}} = 42$, $^{8}p_{r} = 336$; So Find $\underline{|n-r}$

Prove each of the following

$$^{11}p_3 = \frac{\lfloor 11}{\lfloor 8}$$

$$^{20}p_4 = \frac{\lfloor 20}{\lfloor 16}$$

we deduce . $\boxed{^{n}p_r = \frac{\lfloor n}{\lfloor n-r}}$

N . B : $\lfloor 0 = 1$, $^{n}P_0 = 1$

Ex: $^{n}p_2 = 30$ *find the value of n*

Ex: If $^{n}P_5 = 90 \times {}^{n-2}P_3$ *Find the value of n*

Ex: If $^{10}P_r = 30\,240$ *Find r*

Find S . S $\lfloor n+1 : \lfloor n-1 = 27$

Ex : prove that $\;{}^{n}P_{r} : {}^{n-1}P_{r-1} = n$

Ex : If $\dfrac{1}{\lfloor\underline{n-1}} + \dfrac{2}{\lfloor\underline{n}} = \dfrac{210}{\lfloor\underline{n+1}}$ *find n*

Ex : If ${}^{n}p_{r} = 2520$ *and* $\lfloor\underline{r} = 120$ *. Find* $\lfloor\underline{3r - 2n}$

Each set, containing some or all the elements of a set without considering of the order of elements.

$$^{n}C_{r} = \frac{^{n}p_{r}}{\lfloor r}, \quad n \geq r \quad \text{and} \quad n, r \in \mathbb{N}$$ <u>numerical Form</u>

$$^{n}C_{r} = \frac{\lfloor n}{\lfloor n-1 \, \lfloor r} \qquad n \geq r \; ; \; n, r \in \mathbb{N}$$ <u>Mathematical form</u>

Important corollaries

$$^{n}C_{r} = {}^{n}C_{n-r} \qquad , \frac{1}{2}n > r$$

$$^{n}C_{n} = {}^{n}C_{0} = 1$$

If $^{n}C_{r} = {}^{n}C_{x}$, so either r = x or r + x = n

$$^{n}C_{1} = 1$$

$$\frac{^{n}C_{r}}{^{n}C_{r-1}} = \frac{n-r+1}{r}$$

Ex : Find S . S of each of the following

$^{x}C_{2} = 6$ $\qquad$ $^{x-3}C_{3} = 35$

Ex : If $^{n}C_{r} = 1$, $^{n+3}C_{r+1} = 28$ find values of n and r .

$Ex: Find\ S.S \quad : {}^{2x+1}C_{x+5} = {}^{2x+1}C_{11}$

$Ex: if\ {}^{n}p_{r} : {}^{n}C_{r} = 720 \quad , \quad {}^{n+1}C_{r+1} = {}^{n+1}C_{r-3} \quad . Find\ {}^{n}C_{n-2}$

Ex: If ${}^{n}C_{r+1} : {}^{n}C_{r-1} = 7 : 5$, ${}^{n}C_{r} = {}^{n}C_{r+1}$. Find the value of n and r

Ex: Solve each of the following : -

${}^{n}C_{13} : {}^{n}C_{12}$

${}^{n}C_{n-4} = 120$

Prove that : ${}^{n-1}C_{m} + {}^{n-1}C_{m+1} = {}^{n}C_{m+1}$.

If ${}^{n}C_{n-2} = 55$ *. find* n

If ${}^{18}C_{3r-7} = {}^{18}C_{2r+5}$ Find r

If ${}^{n}C_{10} > {}^{n}C_{9}$. Prove that $n > 19$